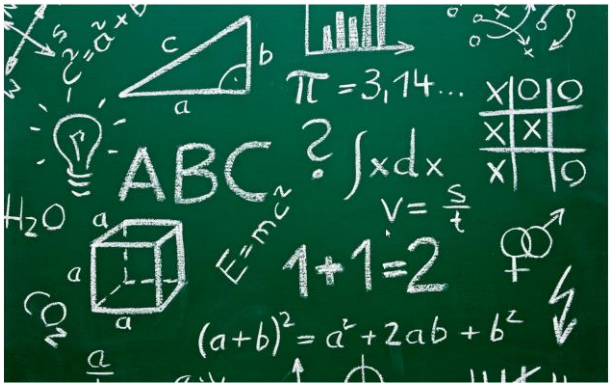
Math Review

Updated 8/2022



Limited Treatment Short Course

MICHIGAN DEPARTMENT OF
ENVIRONMENT, GREAT LAKES, AND ENERGY

Fractions

Basics of fractions: $\frac{3}{5}$ ← numerator (3), ← denominator (5)

<u>3 Important Rules about Fractions:</u>

1. Fractions represent a division problem top divided by the bottom.

Ex. 3 ÷ 5 = 0.6

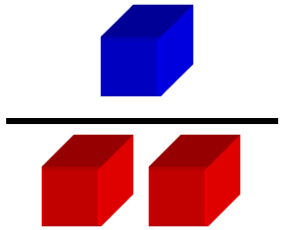
2. Any whole number is a fraction because it is always over 1. Any number divided by 1 is that number.

$$\frac{7}{1} = 7 \qquad \frac{9}{1} = 9 \qquad \frac{22}{1} = 22$$

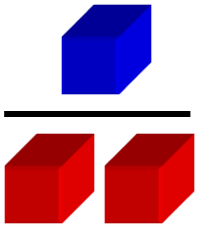
3. Any number or unit over itself equals 1. Essentially it cancels itself.

$$\frac{9}{9} = 1 \qquad \frac{12}{12} = 1 \qquad \frac{\text{ft}}{\text{ft}} = 1 \qquad \frac{\text{mile}}{\text{mile}} = 1$$

Ratios & Proportions



The ratio of blue cubes to red cubes is: $\frac{1}{2}$



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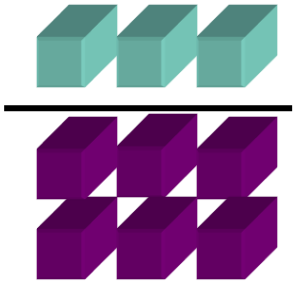


OR $\frac{1}{2} = \frac{3}{6}$

The ratio of teal cubes to purple cubes is equal to the ratio of blue cubes to red cubes.

This relationship is called a proportion.

Fractions

$$\text{Example: } \frac{1}{2} + \frac{8}{12} + \frac{12}{16} + \frac{3}{4}$$

Whenever you see fractions convert to decimals by doing the divisional math, and then do the rest of the math.

1 ÷ 2 = 0.5
8 ÷ 12 = 0.67
12 ÷ 16 = 0.75
3 ÷ 4 = 0.75

0.5 + 0.67 + 0.75 + 0.75 = 2.67

Decimals & Percentages

Fraction	Decimal	Percent
1	1.0	100%
1/2	0.5	50%
1/3	0.33	33.3%
1/4	0.25	25%
1/5	0.2	20%
1/6	0.166	16.6%
1/8	0.125	12.5%
1/10	0.1	10%
1/100	0.01	1%

Fractions, decimals, and percentages are all related to one another as shown in the table above each other.

Decimals & Percentages

Fraction	Decimal	Percent
1	1.0	100%
1/2	0.5	50%
1/3	0.33	33.3%
1/4	0.25	25%

Percentages when used in math not as a description need to be converted to a decimal in order to be properly used. Conversion is shown below…

Converting Percent to Decimal by simply moving the decimal point 2 places to the left.

Converting Decimal to Percent by simply moving the decimal point 2 places to the right.

Examples:

75% = 0.75 0.09 = 9% 57% = 0.57 1.27 = 127%

Decimals & Percentages

- Determining how much of something is within a container based on a percentage is often practiced in the water field.

Example:

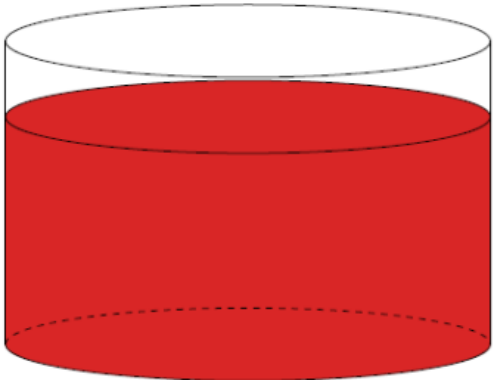
- What is 25% of a 55 gallon drum?
  - As mentioned above convert all percentages to a decimal in order to perform the math functions, 0.25.
    - 0.25 x 55 gallons = 13.75 gallons
- "Percent of" is the same as saying multiply by, and you multiply by the number we want the percentage of.

Decimals & Percentages

- Determining how full a chemical feed tank with solution is left in it based on a percentage helps understanding how much is left.

Example:

- A 500 gallon chlorine solution tank has 320 gallons in it. What percent of the tank is full?
- Remember: $\frac{\text{Part of}}{\text{whole}} = \frac{320}{500} = 0.64$



- Now convert the decimal to percent,
  - $0.64 \times 100 = 64\%$

Quick Review

Convert the decimals to percentages.

1. 0.63 =
2. 1.12 =
3. 0.74 =
4. 0.004 =

Convert the percentages to decimals.

5. 7% =
6. 41% =
7. 77% =
8. 0.9% =

Perform the following math calculations.

9. A water treatment plant with a capacity of 550,000 gal/day is operating at 75% capacity. How much water is this plant processing?

10. City Hall conducted a survey of the water customers. 2275 out of 3500 people want a new Iron removal plant. What percent of those surveyed want a new iron removal plant?

Quick Review Answers

Convert the decimals to percentages.

1. 0.63 = 63%
2. 1.12 = 112%
3. 0.74 =74%
4. 0.004 = 0.4%

Convert the percentages to decimals.

5. 7% = 0.07
6. 41% = 0.41
7. 77% = 0.77
8. 0.9% = 0.009

Perform the following math calculations.

9. A water treatment plant with a capacity of 550,000 gal/day is operating at 75% capacity. How much water is this plant processing?

$$convert\ 75\%\ to\ decimal = 0.75$$

$$550{,}000\,\frac{gal}{day}\,x\ 0.75 = 412{,}500\,\frac{gal}{day}$$

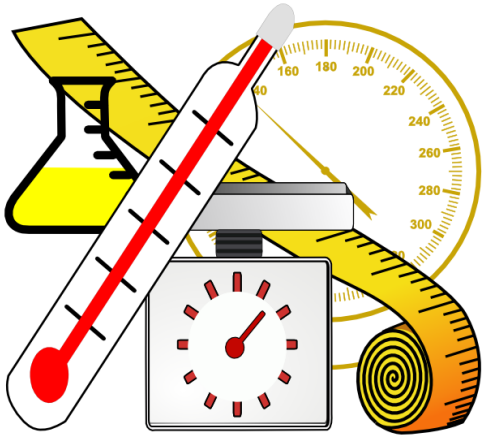
10. City Hall conducted a survey of the water customers. 2275 out of 3500 people want a new Iron removal plant. What percent of those surveyed want a new iron removal plant?

$$\frac{Part}{Whole}\ x\ 100 = \frac{2275}{3500}\,x\ 100 = 65\%$$

Unit Conversions

This is the most common practice concept that is used in the water industry, but it is also the most crucial for operators to understand.

Numbers are just numbers, but when you attach units to them they become vastly different.



Practicality now meets numbers:

1 foot = 12 inches
1 mile = 5280 feet
1 yard = 3 feet = 36 inches

Note-when performing addition, subtraction, multiplication, or division to figures, all the units must be the same.

Unit Conversions

1 foot = 12 inches

$$\frac{1\ foot}{12\ inches} \text{ or } \frac{12\ inches}{1\ foot}$$

$$\frac{1\ \cancel{foot}}{12\ \cancel{inches}} \times \frac{12\ \cancel{inches}}{1\ \cancel{foot}} = 1$$

- The above math problem shows that inverse fractions (one is upside down compared to the other) are equal to each other.
  - The units cancel each other out
- Example: Convert 2.2 feet to inches.

$$\frac{2.2\ \cancel{\text{ft}}}{1} \times \frac{12\ \text{inches}}{1\ \cancel{\text{ft}}} = 26.4\ \text{inches}$$

Unit Conversions

Example: How far is 8 miles, 50 yards and 4 feet, in feet?

Start by converting all of the values to feet, then add them together:

$$8 \text{ miles} = \frac{8 \cancel{\text{miles}}}{1} \times \frac{5{,}280 \text{ feet}}{1 \cancel{\text{mile}}} = 42{,}240 \text{ feet}$$

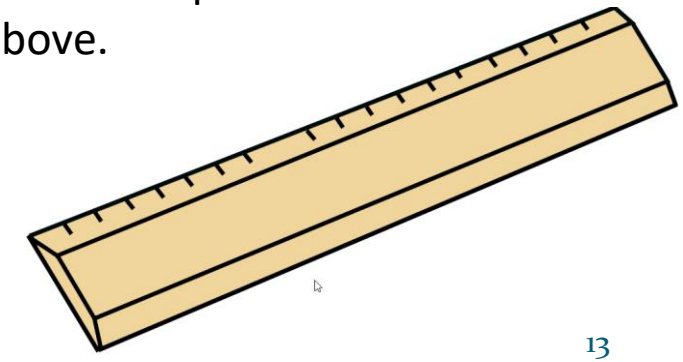
$$50 \text{ yards} = \frac{50 \cancel{\text{yards}}}{1} \times \frac{3 \cancel{\text{feet}}}{1 \text{ yard}} = 150 \text{ feet}$$

$$4 \text{ feet} = 4 \text{ feet}$$

Add 42,240 feet, 150 feet, and 4 feet together:

$$42{,}240 \text{ feet} + 150 \text{ feet} + 4 \text{ feet} = 42{,}394 \text{ feet}$$

When similar units are on top and bottom of the fractions even if separated from one another via multiplication the units cancel as shown above.

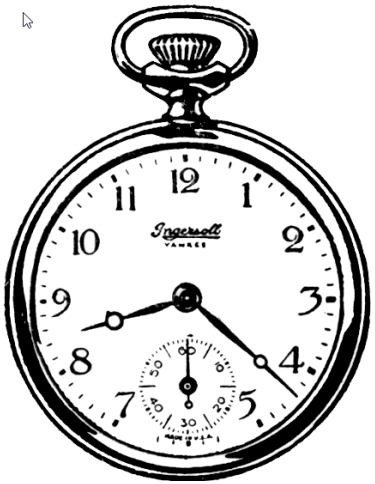


Unit Conversions

Rule: Multiply everything across the top, and everything across the bottom, and divide at the end if needed.

Example: Convert 1 day to seconds:

$$\frac{1\text{ day}}{1} \times \frac{24\text{ hours}}{1\text{ day}} \times \frac{60\text{ min}}{1\text{ hours}} \times \frac{60\text{ sec}}{1\text{ min}} = \frac{(1\ \cancel{\text{day}} \times 24\ \cancel{\text{hours}} \times 60\ \cancel{\text{min}}\ \times 60\text{ sec})}{(1\ \times 1\ \cancel{\text{day}} \times 1\ \cancel{\text{hour}} \times 1\ \cancel{\text{min}})} = \frac{86{,}400\text{ sec}}{1}$$

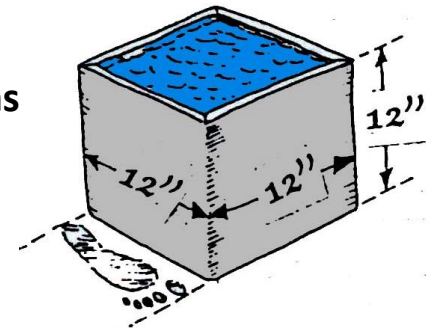


Unit Conversions

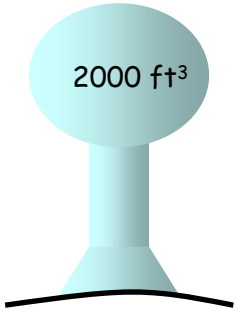
Common Water Units:

1 gallon = 8.34 pounds **1 cubic foot (ft^3) = 7.48 gallons**

1 psi = 2.31 feet of water **1 foot of water = 0.433 psi**



Example: A storage tank holds 2,000 cubic feet of water.
a.) How many gallons of water will it hold?
b.) How much does that water weigh?



$$\frac{2000\cancel{ft^3}}{1} \times \frac{7.48 \text{ gallons}}{1\cancel{ft^3}} = 14{,}960 \text{ gallons}$$

$$\frac{14{,}960 \text{ gall}\cancel{\text{ons}}}{1} \times \frac{8.34 \text{ pounds}}{1 \text{ gall}\cancel{\text{ons}}} = 124{,}766.4 \text{ pounds of water}$$

Problems

1. How many pounds does five cubic feet of water weigh?

2. How many cubic feet does 13 gallons of water occupy?

3. A tank has a capacity of 90,000 cubic feet. What is the capacity of the tank in gallons?

Problems Answers

1. How many pounds does five cubic feet of water weigh?

$$5ft^3 x \frac{7.48\ gal}{1\ ft^3} x \frac{8.34\ lbs}{gal} = 312\ lbs\ water$$

2. How many cubic feet does 13 gallons of water occupy?

$$13\ gal\ x \frac{1\ ft^3}{7.48\ gal} = 1.7\ \mathrm{ft}^3$$

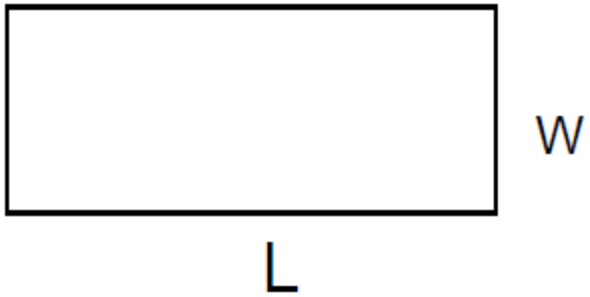
3. A tank has a capacity of 90,000 cubic feet. What is the capacity of the tank in gallons?

$$90{,}000 ft^3\ x\ \frac{7.48\ gal}{ft^3} = 670{,}000\ gallons$$

Areas

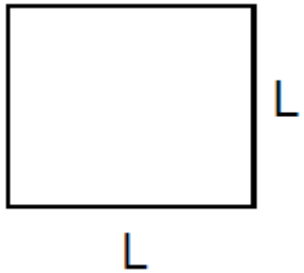
Rectangle

Area = length (L) x width (W)



Square

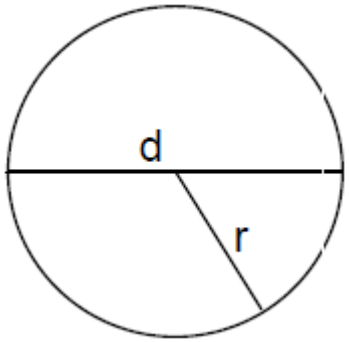
Area = length (L) x length (L)



Circle

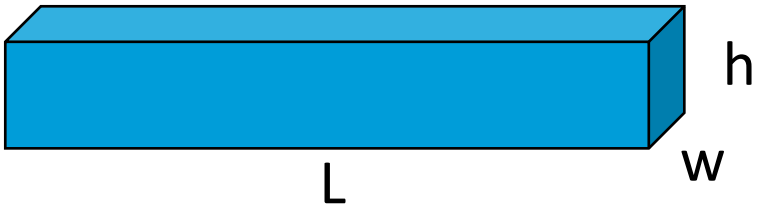
Area = 0.785 x diameter ^{2} = 0.785 (d^2)

Area = 3.14 x radius ^{2} = 3.14 (r^2)



Volumes

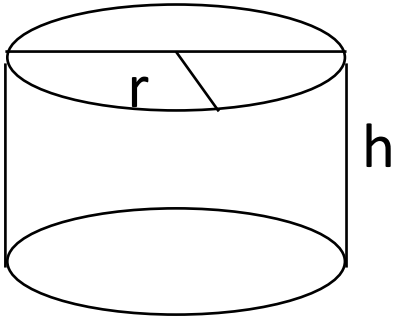
Rectangular Prism:



Volume = area of base (L x w) times height (h)

$V = w \times L \times h$

Cylinder:

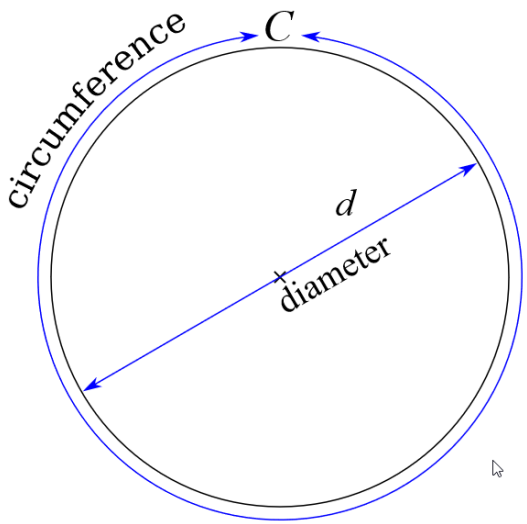


Volume = area of base ($3.14 \times r^2$ or $0.785 \times d^2$) x height (h)

$V = 3.14 \times r^2h = .785 \times d^2h$

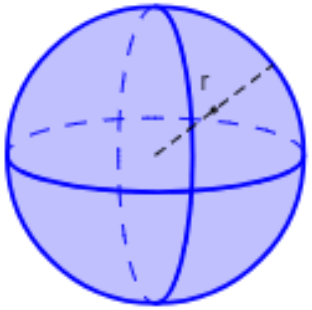
Special Cases

Circumference is not used much but can be helpful when you need to calculate the distance around the outside of the circle.



Circumference (C) = (2 x 3.14 x r) or (3.14 x d)

Example: Well head distance from a source of contamination.



Volume of Sphere

$$= \frac{4}{3}\pi r^3$$

*Remember that the distance from the center point in a sphere is the same in all directions including up and down.

Problem 1

1. Determine the volume of a chemical feed solution tank with a 36-inch diameter and is 4 feet high.

   a. How many gallons will it hold?

   b. What is the total weight, assuming its filled with water?

Problem 2 & 3

2. What is the volume, in gallons, of a water main that is 1000ft long and has a diameter of 12 inches?

3. How many gallons would a rectangular basin measuring 30ft by 16ft by 10ft, hold?

Problems 4, 5, 6

4. What is the volume, in cubic feet, of a 12 inch main that is 1000 feet in length?

5. How many gallons would a rectangular basin, measuring 45 feet by 18 feet by 12 feet, hold?

6. How many hours will it take to fill a ground storage tank that has a diameter of 50 ft and is 60 ft tall with a pump that produces 350 gpm? How long will it take to fill the tank?

Answer to Problem 1

1. Determine the volume of a chemical feed solution tank with a 36-inch diameter and is 4 feet high.

   a. How many gallons will it hold?

$$Convert\ all\ units\ to\ feet:\ \ diameter = 36\ inch\frac{1\ ft}{12\ inch} = 3\ feet$$

$$Calculate\ tank\ volume = 0.785\ h\ d^2 = 0.785\ (4ft)(3\ ft)^2 = 28.26\ ft^3$$

$$Convert\ to\ gallons = 28.26\ ft^3\frac{7.48\ gal}{ft^3} = \mathbf{211\ gallons}$$

   b. What is the total weight, assuming its filled with water?

$$211\ gal\ \frac{8.34\ lbs}{gal} = \mathbf{1760\ lbs\ water}$$

Answers to Problem 2 & 3

2. What is the volume, in gallons, of a water main that is 1000ft long and has a diameter of 12 inches?

$$Convert\ diameter\ to\ inches = 12\ inch \frac{1\ ft}{12\ inch} = 1\ ft$$

$$V = 0.785\ h\ d^2 = 0.785\ (1000ft)(1ft)^2 = 785\ ft^3$$

$$785\ ft^3 \frac{7.48\ gal}{ft^3} = \mathbf{5872\ gallons}$$

3. How many gallons would a rectangular basin measuring 30ft by 16ft by 10ft, hold?

$$V = (L)(W)(H) = (30ft)(16ft)(10ft) = 4800ft^3$$

$$4800ft^3 \frac{7.48\ gal}{ft^3} = 35{,}900\ gallons$$

Answers to Problems 4, 5, 6

4. What is the volume, in cubic feet, of a 12 inch main that is 1000 feet in length?

$$V = 0.785\ h\ d^2 = 0.785\ (1000ft)(1ft)^2 = \mathbf{785\ ft^3}$$

5. How many gallons would a rectangular basin, measuring 45 feet by 18 feet by 12 feet, hold?

$$V = (L)(W)(H) = (45\ ft)(18\ ft)(12\ ft) = \mathbf{9,720\ ft^3}$$

$$9{,}720\ ft^3\ x\ \frac{7.48\ gal}{ft^3} = \mathbf{72,700\ gallons}$$

6. How many hours will it take to fill a ground storage tank that has a diameter of 50 ft and is 60 ft tall with a pump that produces 350 gpm? How long will it take to fill the tank?

$$V = 0.785\ h\ d^2 = 0.785\ (60\ ft)(50\ ft)^2 = 117{,}750\ ft^3\ x\ \frac{7.48\ gal}{ft^3} = 880{,}770\ gal$$

$$Time = \frac{Volume}{Flow} = 880{,}700\ gal\ x\ \frac{min}{350\ gal} = 2516\ min$$

$$2516\ \text{min}\ x\ \frac{hr}{60\ min} = \mathbf{42\ hours}$$

Cross Multiplying

- Cross multiplication is used when solving for an unknown with proportions

$$\frac{a}{b} = \frac{c}{d} \Rightarrow a \times d = b \times c$$

- Example:

$$\frac{1}{2} = \frac{X}{8}$$

$$2 \times X = 8 \times 1$$

$$\frac{2X}{2} = \frac{8}{2}$$

$$X = 4$$

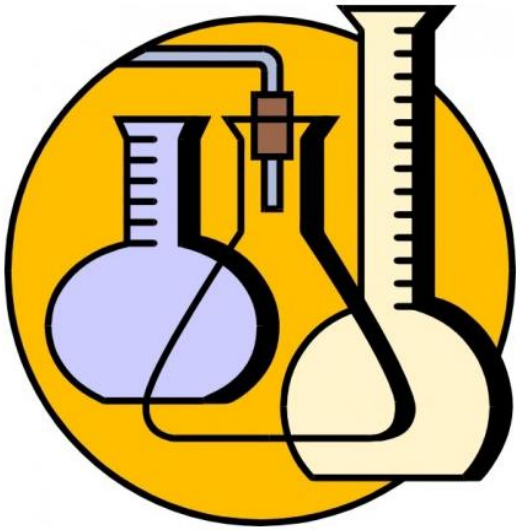
To solve for X (the unknown!)
Cross Multiply and then divide.

Chemical Dosing

- Concentrations are typically expressed in ppm or mg/L
- The term mg/L and ppm are interchangeable

$$\text{ppm} = \frac{\text{pounds of pure chemical}}{\text{million pounds of water}}$$

$$\text{mg/L} = \frac{\text{milligrams of chemical}}{\text{liters of water}}$$



Chemical Dosing

Example: One pound of pure chlorine in 5,000 pounds of water equals, what concentration of mg/L?

$$\frac{5{,}000 \text{ lbs}}{1{,}000{,}000} = 0.005 \text{ million lbs (Mlbs)}$$

$$\frac{1 \text{ lbs of chlorine}}{0.005\text{M lbs } H_2O} = 200\text{mg/L}$$

Note – Whenever dealing with chemical dosing problems it is very important to be sure the water is in pounds and that they are converted to million pounds by dividing it by a 1,000,000.

Chemical Dosing

In most operations, 100% available chemical is not used (gas chlorine is the exception). Frequently, some other concentration of available chlorine is used. These concentrations are expressed in terms of "percent available chlorine." Typical percentages include; 5.25%, 10%, and 65% available chlorine.

Rule of Thumb:

Going from compound to pure, you multiple by the percent.
Going from pure to compound, you divide by the percent.

Chemical Dosing

Example: How many pounds of chlorine are in 4 lbs. of a compound that has 65 % available chlorine?

1) Convert the percent to a decimal = 0.65

2) 4 lbs. of compound x 0.65 = 2.6 lbs. of pure chlorine

Example: If we calculate, we need 9 lbs. of pure chlorine to provide a desired dosage, how many pounds of 65% available chlorine do we need for this?

1) Convert the percent to a decimal = 0.65

2) $\frac{\text{9 lbs. of pure chlorine}}{0.65}$ = 13.85 lbs. of 65% available chlorine

Dosing Problems

1. 5 pounds of chlorine in 7 million pounds of water equals what concentration in ppm?

2. 5 pounds of chlorine in 200,000 pounds of water equals what concentration in ppm?

Dosing Problems

3. 6 pounds of chlorine in 15,000 gallons equals what concentration in ppm?

4. Theoretically, how many pounds of pure chlorine must be added to 900,000 gallons of water to produce a residual of 2.0 ppm?

Dosing Problems

5. How many pounds of calcium hypochlorite (65% available chlorine) would be required to disinfect 800 feet of 8-inch water main with 50 ppm of chlorine?

Answers: Dosing Problems

1. 5 pounds of chlorine in 7 million pounds of water equals what concentration in ppm?

$$ppm = \frac{lbs\ chlorine}{Mlbs\ water} = \frac{5\ lbs\ chlorine}{7\ Mlbs\ water} = \mathbf{0.71\ ppm}$$

2. 5 pounds of chlorine in 200,000 pounds of water equals what concentration in ppm?

$$\frac{200000\ lbs\ water}{1000000\ Mlbs/lbs} = 0.2\ Mlbs\ water$$

$$ppm = \frac{lbs\ chlorine}{Mlbs\ water} = \frac{5\ lbs\ chlorine}{0.2\ Mlbs\ water} = \mathbf{25\ ppm}$$

Answers: Dosing Problems

3. 6 pounds of chlorine in 15,000 gallons equals what concentration in ppm?

$$15000\ gal\ x\ \frac{8.34\ lbs}{gal}\ x\ \frac{Mlbs}{1000000\ lbs} = 0.1251\ Mlbs$$

$$ppm = \frac{6\ lbs\ chlorine}{0.1251\ Mlbs\ water} = \mathbf{48\ ppm}$$

4. Theoretically, how many pounds of pure chlorine must be added to 900,000 gallons of water to produce a residual of 2.0 ppm?

$$900000\ gal\ x\ \frac{8.34\ lbs}{gal}\ x\ \frac{Mlbs}{1000000\ lbs} = 7.506\ Mlbs$$

$$lbs\ chlorine = ppm\ x\ Mlbs\ water = 2\ x\ 7.506 = \mathbf{15\ lbs\ chlorine}$$

Answers: Dosing Problems

5. How many pounds of calcium hypochlorite (65% available chlorine) would be required to disinfect 800 feet of 8-inch water main with 50 ppm of chlorine?

$$d = 8\ inch\ x\ \frac{1\ ft}{12\ inch} = 0.67\ ft$$

$$V = 0.785\ h\ d^2 = 0.785(800ft)(0.67ft)^2 = 281.9\ ft^3$$

$$281.9ft^3\ x\ \frac{7.48\ gal}{ft^3}\ x\ \frac{8.34\ lbs}{gal}\ x\ \frac{Mlbs}{1000000lbs} = 0.0176\ Mlbs\ water$$

$$lbs\ chlorine = ppm\ x\ Mlbs\ water = 50\ x\ 0.0176 = 0.88\ lbs$$

$$lbs\ chemical = \frac{lbs\ chlorine}{percent\ available\ chlorine} = \frac{0.88\ lbs\ chlorine}{0.65}$$

$$= \mathbf{1.35\ \boldsymbol{lbs\ calcium\ hypochlorite}}$$

Questions?