Climate Risk Disclosure and Risk Sharing in Financial Markets*

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Abstract

In this paper, I model how climate risk disclosure affects investors' ability to use the financial market to share climate risk. In stark contrast to the Hirshleifer effect, when investors face short-sale constraints, I show that climate risk disclosure can render financial markets *more* effective at enabling risk sharing. The reason is that precise knowledge of firms' climate exposures is essential for investors to form efficient climate-hedging portfolios. I characterize when firms' incentives to voluntarily disclose their climate risks are aligned with social efficiency and thus when disclosure mandates can enhance risk sharing. Finally, I show that climate risk disclosure alters the climate risk premium and can lower the valuations of moderately green firms, thereby transferring wealth between brown and green investors.

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Keywords: climate finance, climate risk disclosure, carbon disclosure, investor welfare, disclosure regulation, risk sharing, Hirshleifer effect

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1 Introduction

Across the globe, regulatory authorities are considering or actively imposing requirements that firms disclose information on their climate-related risks and greenhouse gas emissions. Furthermore, Ilhan, Krueger, Sautner, and Starks (2023) present survey evidence showing that a majority of global institutional investors "believe climate risk reporting to be at least as important as financial reporting, with almost one-third considering it to be more important." However, existing theory is unclear on the welfare impact of climate risk disclosure. A distinctive feature of climate-related risks is that investors have diverse preferences for and exposures to these risks, and trade on these preferences and exposures in financial markets. Moreover, models of trade among diverse investors often suggest that more public information *decreases* investor welfare (Hirshleifer, 1971). The reason is that public information can resolve risks before investors have the opportunity to trade.

While this may suggest that climate risk disclosure mandates adversely affect markets, models of disclosure and risk sharing focus on information that concerns the *outcome* of a specific risk. For instance, applied to climate risk, these models address the impact of information regarding the severity of climate change. However, firms are unlikely to possess information on climate outcomes, which lie in climate scientists' domain of expertise. Instead, firms possess information on their *exposures* to climate outcomes. For example, firms' information on their carbon emissions may be useful in predicting their exposures to regulatory shifts that may accompany adverse climate outcomes, and firms' information on their supply chains may help assess how robust they are to adverse weather events.

This paper shows that disclosure regarding firms' risk exposures does not harm, but instead enhances investors' ability to share risk in the financial market. The mechanism behind this finding is simple, and yet absent from existing theory: in order to effectively trade on their preferences or exposures in the stock market, investors need to understand firms' risk attributes. For example, consider investors who wish to hedge climate risk or directly obtain utility from holding "green" stocks. Climate risk disclosures may help investors identify the stocks that contribute most to mitigating climate change, or that perform best following adverse climate shocks. This, in turn, enables them to identify the ideal set of stocks to purchase or sell.[1] The importance of this channel is underscored by the survey evidence in Krueger, Sautner, and Starks (2020), who find that "... many market participants, including institutional investors, find climate risks difficult to price and hedge, possibly because of their systematic nature, a lack of disclosure by portfolio firms, and challenges in finding suitable

[1] While I focus on climate risk disclosure because it is an important application and helps to guide the analysis, most of the results in the paper apply to disclosure about more general firm risk exposures that investors seek to trade on in the financial market.

hedging instruments."

I investigate this mechanism and its implications in a model where "green" and "brown" investors trade in a continuum of stocks. Certain firms are "greener," or less exposed to the climate, than others – that is, on a relative basis, they perform well following adverse climate news. Investors and firms do not know the firms' precise climate exposures, but each firm privately observes information on its exposure (e.g., its carbon emissions and supply chain). Following the literature that studies the Hirshleifer effect, firms disclose prior to trade, but novel to this literature, their disclosures concern their risk exposures rather than their expected future cash flows. In my baseline model, I assume that green and brown investors have different exposures to climate risk, which captures geographical differences, differences in age, or holdings of climate-exposed assets like real estate. However, I demonstrate that similar results hold when the gains from trade between green and brown investors stem from divergent preferences rather than risk exposures.

I first study the standard "frictionless" benchmark applied in disclosure theory in which investors can costlessly take long or short positions (e.g., Diamond, 1985). In this case, I find that even a trivial amount of information on firms' climate exposures is sufficient for investors to trade to an efficient allocation of climate risk. Intuitively, green investors purchase well-diversified portfolios that are long green firms and short brown firms, and brown investors take the other side of this trade. When there is a large amount of uncertainty over the greenness of any given firm, the climate exposure of each dollar invested into such portfolios is minimal. However, as these portfolios are well diversified, the investors can lever them (and hedge any resulting market risk) until their climate risk exposures are fully shared. Thus, precise disclosure on climate risk exposures is not necessary to achieve risk-sharing efficiency (but, in contrast to the Hirshleifer effect, also does not harm such efficiency).

While this result is theoretically appealing, it rests on investors' ability to take short positions. In particular, when there are few green firms that are potent climate hedges available in the market, to efficiently share risk, green (brown) investors must hold potentially large short positions in brown (green) firms. This behavior does not match the observed portfolios of many investors. In reality, institutional constraints often prevent investors from taking large short positions, observed short interest is a small percentage of overall market capitalization, and ESG funds often take only long positions. I thus next re-consider the role of climate risk disclosure when investors are constrained to hold only long positions.[2] In this case, investors' only means to hedge climate risk is to tilt their portfolios away from brown

[2]While I assume a complete inability to short in the baseline model for simplicity, I show that the general insights continue to hold when the supply of shares that can be shorted is capped. However, the equilibrium will always ultimately be efficient as this cap grows large, even with only minimal climate risk disclosure.

stocks and towards green stocks.

I show that the equilibrium that arises under short-sales constraints depends critically on the informativeness of firms' climate risk disclosures. When these disclosures are relatively uninformative, short-sale constraints bind in equilibrium and risk is not shared efficiently. Intuitively, in this case, investors struggle to identify stocks that have significantly positive or negative climate exposures. Nevertheless, in equilibrium, green investors do their best to hedge climate risk by purchasing the stocks that are expected to have the lowest climate exposures and "divesting" entirely from stocks that are expected to have the highest exposures. This is consistent with the common institutional practice of excluding stocks with specific business models or low climate scores, but not short selling these stocks, which is difficult to explain using models that lack portfolio constraints (e.g., Pástor, Stambaugh, and Taylor, 2021).

Green stocks are priced at a premium in equilibrium because they hedge climate risk. This leads brown investors to exclude sufficiently green stocks from their portfolios. In sum, the market is segmented: brown investors own brown stocks and green investors own green stocks. Yet, green investors' portfolios do not fully hedge their climate exposures, and one of the investor groups holds too much market risk relative to the social optimum. More precise disclosure remedies this problem by enabling the green investors to accurately identify and purchase stocks that have low climate exposures. When there is sufficient cross-sectional variation in firms' climate exposures and climate risk disclosure is sufficiently precise, short-sale constraints do not bind and investors efficiently share risk in equilibrium. Put simply, disclosure enables investors to form efficient hedging portfolios without the need to short.

I next study the role of disclosure mandates in improving risk-sharing efficiency. For a disclosure mandate to be useful, managers' incentives to *voluntarily* disclose their climate exposures must be too small relative to the social benefits of such disclosures. To assess when this is the case, I endogenize climate risk disclosure in the model by assuming firms are run by managers who observe their climate exposures and can verifiably disclose them to the market. Following the voluntary disclosure literature, I consider two frictions that can prevent managers from fully revealing their information. First, as in Verrecchia (1983), I consider disclosure costs, which may capture, for instance, the costs to verifying information on carbon emissions or the proprietary costs to revealing firms' supply chains. Second, as in Dye (1985), I consider uncertainty over managers' information endowments, which may capture uncertainty over whether managers have collected information on their emissions.

When climate disclosure is cheap but managers may be uninformed, I show that firms' incentives to voluntarily disclose are often just as potent as disclosure mandates in achieving risk-sharing efficiency. The idea is that, in a voluntary disclosure equilibrium, firms with low climate exposures disclose these exposures, while firms with high climate exposures do not. A disclosure mandate alters this equilibrium by forcing firms with high climate exposures to also disclose. However, this is often not necessary for investors to be able to share climate risk.

Intuitively, as long as green investors can identify a portfolio of green firms, brown investors can simply hold the rest of the market. Even though the brown investors face uncertainty over which of their holdings are more exposed to climate risk, the overall allocation of climate risk across investors remains efficient. In contrast, when disclosure is costly, firms may not disclose even when their climate exposures are relatively low. In this case, regulation can be necessary to achieve efficiency, but also directly imposes costs on firms. Whether regulation is optimal depends on how much of these costs reflect deadweight losses of producing and verifying information vs. proprietary costs that simply shift resources across firms.

While my focus is on climate risk disclosure and risk sharing, I show that, when short-sales constraints bind and the market is segmented, the model also delivers novel implications for how firms' disclosures impact their prices. As in Lintner (1969), in the model, firms' prices reflect only the preferences and exposures of the investors who hold them in equilibrium. I show that this leads to a stronger relationship between firms' climate betas and their prices (and thus expected returns) among the firms held by green investors than those held by brown investors. Since green investors hold greener firms, this causes prices to be convex in firms' climate exposures, which raises overall market valuations. The informativeness of firms' climate risk disclosures determines the extent of this convexity, and thus aggregate market valuations. Furthermore, more informative climate risk disclosures can *lower* the valuations of moderately green firms by reducing the climate exposures of the green investors who hold them in equilibrium. Thus, such disclosures need not lower the cost of capital for all green firms.

I extend the model in several ways to illustrate the robustness of the results. First, I show that the results continue to hold when investors can short, as long as the total number of shares available to short is not too large. Second, I show that similar results arise when green and brown investors have the same climate exposures but differ in their preferences for holding green and brown stocks. Finally, I show that the results continue to hold in a setting where markets are not fully segmented because a subset of investors invest in broad-market index funds that do not consider valuations or climate exposures.

The paper proceeds as follows. Section 2 reviews the related literature. Section 3 lays out the model. Section 4 derives the model's equilibrium. Section 5 studies how disclosure informativeness affects risk-sharing efficiency, and studies investor welfare and stock prices.

Section 6 considers the role of disclosure mandates by modeling firms' incentives to voluntarily disclose their climate risk exposures. Section 7 demonstrates the robustness of the findings. Section 8 summarizes the empirical predictions and Section 9 concludes. Appendix A contains all proofs of the formal results.

2 Literature Review

My paper contributes to the literatures on public information and risk sharing, risk disclosures, non-financial preferences and disclosure, and portfolio constraints.

Public Information and Risk Sharing. Hirshleifer (1971) demonstrates that, in pure-exchange models, pre-trade public information resolves uncertainty before investors have an opportunity to share risk, which lowers investor welfare. This mechanism has motivated considerable research across many fields, including banking, insurance, and equity markets.

When applied to financial markets, it is often argued that the Hirshleifer (1971) effect is muted because investors have frequent opportunities to trade and so should always be able to trade prior to a public information release. This argument follows from Milgrom and Stokey (1982)'s no-trade theorem, which implies that, in a complete market, a public information signal has no impact on risk-sharing efficiency when investors can trade prior to its release.[3] However, the Hirshleifer (1971) effect should hold whenever disclosure events occur prior to trade among investors. Thus, the fact that trade occurs continuously, including after most information releases (in violation of the no-trade theorem), suggests that the Hirshleifer (1971) effect may in fact play an important role in financial markets. Relatedly, investors only enter markets gradually over their lifetimes, and prefer less public information before they enter the market. This force is studied in overlapping-generations models (Gao, 2010; Dutta and Nezlobin, 2017).[4]

My paper contributes to this work by showing that information on risk exposures can enhance the ability of investors to share risk in the presence of short-sale constraints, even when it arrives prior to investors' ability to trade. The role of short-sale constraints in moderating risk disclosure's impact on welfare appears unique to such disclosure: while the literature on the Hirshleifer effect does not consider portfolio constraints, qualitatively, the findings from this work do not appear to rely on a lack of such constraints. Importantly, the

[3]Formally, because the initial allocation is Pareto efficient and the disclosure does not lead to trade in their setting, it has no impact on risk-sharing efficiency. It is also important for their result that investors have "concordant beliefs," i.e., they agree on the signal's distribution given the fundamental. See also Marshall (1974) and Hakansson, Kunkel, and Ohlson (1982).

[4]A countervailing force is that investors who are already in the market and must sell, for instance due to a need to liquidate assets to consume in retirement, prefer more disclosure prior to exiting the market.

mechanism I study would continue to arise even if investors could trade before the disclosure. Milgrom and Stokey (1982)'s no-trade theorem does not apply in my setting because markets are incomplete, so that investors cannot arrive at Pareto-efficient risk allocations prior to the disclosure, and the disclosure influences trade among investors.

Other work has documented several alternative channels through which disclosure may affect investor welfare both in pure-exchange models and in models with investment. Goldstein and Leitner (2018) show that some level of disclosure can be essential to risk sharing in a banking setting. Diamond (1985) shows that disclosure can reduce costly, duplicative private information acquisition and prevent such information from leading to inefficient risk sharing. Dow and Rahi (2003) show that information on a component of cash flows that is orthogonal to a risk that investors seek to hedge increases risk-sharing efficiency. Their mechanism relies on public information increasing the correlation between the firm's cash flows and the hedgable risk, while, in my setting, public information instead enhances risk-sharing efficiency by providing information on this correlation. Moreover, disclosure can influence production via several mechanisms (see the reviews by Dye, 2001; Goldstein and Yang, 2017; Kanodia and Sapra, 2016).

Risk Disclosure. The effects I study apply more generally to disclosures over firms' exposures to risks that investors have different exposures to or tastes for, and seek to trade on in stocks. Other work has considered the effects of risk disclosures on market outcomes, but does not study risk sharing. Jorgensen and Kirschenheiter (2003) analyze the equilibrium that arises when firms can voluntarily disclose about their risk. Heinle, Smith, and Verrecchia (2018) study the asset-pricing implications of risk-exposure disclosure, and Smith (2022) studies how risk disclosure affects information acquisition and feedback from prices to investment decisions. Schmalz and Zhuk (2019) show that investor learning about firms' risk exposures from earnings generates heightened volatility in downturns and skewness in returns. Studying risk disclosure and risk sharing requires a model where investors have heterogeneous exposures to the underlying risk factor, which departs from the homogeneous investor framework taken in the existing work.

Non-Financial Preferences. A growing body of work demonstrates that investor preferences for non-financial attributes of firms can alter asset prices, trading patterns, and the role of corporate disclosure. In models without portfolio constraints, Pástor et al. (2021) show that green assets have lower costs of capital because of investors' preferences for and exposures to climate risk, and Pedersen, Fitzgibbons, and Pomorski (2021) show that ESG preferences generate a four-factor efficient frontier. In related settings, Friedman, Heinle, and Luneva (2021) and Chen and Schneemeier (2023) consider the role of greenwashing, and Aghamolla and An (2023) and Xue (2023) study how ESG disclosure affects investment efficiency. Berk and Van Binsbergen (2021) show that divestment strategies have minimal impact on expected returns. Consistent with this, in my model, divestment does not directly generate significant pricing differences. Instead, these differences arise because aggregate climate risk is priced as a risk factor. Similar to the trading equilibrium in my model, Piccolo, Schneemeier, and Bisceglia (2023) show that partial market segmentation can arise in a setting with ESG investors who face short-sale constraints and are risk neutral but incur quadratic trading costs.

More directly related to my model, Friedman and Heinle (2016) study a setting where some investors derive non-monetary utility from investing in companies with a non-financial attribute, and show that non-financial public information leads investors to adjust their holdings based upon their preferences. Goldstein, Kopytov, Shen, and Xiang (2022) considers the impact of public ESG information on trading and price informativeness in a model with informed investors. Similar to my model, they show that ESG information can *increase* firms' costs of capital, though their mechanism differs. Specifically, their result follows from how such information affects the intensity with which traders speculate on private information. I contribute to this work by considering disclosure on uncertain risk exposures rather than a non-financial payoff, by studying the implications for efficient risk sharing, and by considering voluntary disclosure. I further consider a multiple-asset model, which is essential to uncover the role of short-sale constraints in moderating climate risk disclosure's impact on risk-sharing efficiency.

Portfolio Constraints. My paper further relates to prior work that studies the impact of portfolio constraints, including margin and short-sale constraints, on asset prices and trade (e.g., Lintner, 1969; Diamond and Verrecchia, 1987; Garleanu and Pedersen, 2011; Banerjee and Graveline, 2013; Glebkin, Gondhi, and Kuong, 2021; Nezafat and Schroder, 2022). While such constraints are often left out of disclosure models for tractability, this work suggests that these constraints play a central role in markets. My paper shows how accounting for short-sales constraints can be critical to understanding how disclosure affects investor welfare. Lintner (1969)'s model of the financial market with short-sales constraints is closest to mine, though he focuses on a finite number of stocks with known risk exposures and does not explicitly characterize the equilibrium. By considering a continuum of stocks, I am able to both allow for uncertainty over risk exposures and derive explicit results on equilibrium prices and holdings.

3 Model

I consider a three-period model with $t \in \{0, 1, 2\}$ in which investors trade in the stocks of a continuum of firms. Investors face uncertainty over these stocks' idiosyncratic cash flows and exposures to a systematic climate risk factor. On date 0, firms reveal information signals regarding their climate exposures. On date 1, investors trade in the stocks and on date 2, all random variables are realized and firms pay off their cash flows to shareholders.

Firm cash flows and climate risk. There is a continuum of firms indexed by $j \in [0, 1]$. Firm j's cash flows per share take the following form:

$$\tilde{x}_j = \overbrace{\tilde{\alpha}_j}^{\substack{\text{firm-specific} \\ \text{cash flows}}} + \overbrace{\tilde{\beta}_j}^{\text{climate beta}} \underbrace{\tilde{F}_C}_{\text{climate risk}} + \overbrace{\tilde{F}_M}^{\substack{\text{non-climate} \\ \text{market risk}}}.$$

All random variables are assumed independent. The term $\tilde{F}_C$ represents systematic climate risk, and I assume that $\tilde{F}_C \sim N(\mu_C, \sigma_C^2)$. As Giglio, Kelly, and Stroebel (2021) and Ilhan et al. (2023) discuss, climate risks fit into two categories: (i) physical risks, which capture actual climate outcomes such as rising temperatures, and (ii) transition risks, which capture changes in regulation and demand that result from news on the climate. One can interpret $\tilde{F}_C$ broadly as encompassing either type of risk. The term $\tilde{\alpha}_j$ represents firm j's idiosyncratic payoffs and the term $\tilde{\beta}_j$ captures its exposure to climate risk, or its "climate beta." I let $\tilde{\alpha}_j$ and $\tilde{\beta}_j$ have arbitrary distributions subject to the applicability of the law of large numbers (i.e., they must have finite variances).

For simplicity, I normalize the mean of $\tilde{\alpha}_j$ to zero, but let the mean of $\tilde{\beta}_j$, $\mu_\beta \equiv \mathbb{E}\left[\tilde{\beta}_j\right]$, be positive. This ensures that lower values of $\tilde{F}_C$ represent negative climate news and so reduce the average firm's cash flows. I let $\mu_C \leq 0$ in order to capture the fact that climate change is expected to harm the average firm's future cash flows. Finally, $\tilde{F}_M \sim N(\mu_M, \sigma_M^2)$ captures variation in the value of the market portfolio driven by forces other than the climate. I refer to this as market risk moving forward, with the caveat that it explicitly refers to the component of market risk that is orthogonal to climate risk.

While, for sake of parsimony, I assume that investors have identical priors about each firm's cash flows, the results are qualitatively similar when investors' priors differ across firms. Likewise, the assumption that all firms have identical exposures to $\tilde{F}_M$ is not essential but avoids excess notation. In particular, if firms have different exposures to $\tilde{F}_M$, the results continue to hold upon interpreting $\tilde{\beta}_j$ as the firm's exposure to the climate relative to its exposure to other sources of market risk (i.e., its climate beta divided by its market beta).

Investors. There is a unit continuum of investors that consists of two identical groups. A fraction λ_G of the investors are "green" and a fraction $\lambda_B = 1 - \lambda_G$ are "brown;" I represent these groups in the formal notation using subscripts G and B. Both groups of investors have CARA utility with risk aversion ρ, i.e., an investor with terminal wealth w_i has utility $u(w_i) = -\frac{1}{\rho}\exp(-\rho w_i)$. Green investors have high outside exposures z_G, while brown investors have low outside exposures $z_B < z_G$, to the climate risk factor. Moreover, each group of investors is endowed with κ shares of each stock. Formally,

$$w_G = \int_0^1 D_{Gj}(\tilde{x}_j - P_j)\,dj + \kappa \int_0^1 P_j dj + z_G \tilde{F}_C;$$
$$w_B = \int_0^1 D_{Bj}(\tilde{x}_j - P_j)\,dj + \kappa \int_0^1 P_j dj + z_B \tilde{F}_C,$$

where D_{Gj} and D_{Bj} are the green and brown investors' demands for stock j, respectively, and P_j is firm j's price. I let $\bar{z} = \lambda_G z_G + \lambda_B z_B$ denote the average climate exposure across the investors, and assume that $\bar{z} > 0$. For technical convenience, I further focus on equilibria where investors in each group submit the same demands, which arise naturally given that they are ex-ante identical.[5]

This set up nests as a special case the situation in which brown investors optimize purely over financial profits and ignore firms' climate exposures (except to the extent they influence prices), which can be captured by setting $z_B = 0$. The difference in investors' outside exposures follows Pástor et al. (2021), and the potential for this to lead to gains from trade in climate-exposed stocks is analogous to Banerjee, Breon-Drish, and Smith (2023).[6] Alternatively, another literature models investors as obtaining private benefits from holding certain stocks. In Section 7.2, I show that similar results obtain in this alternative case.

Investors have access to a risk-free bond that is in unlimited supply and has net return normalized to zero. I assume in my primary analysis that these investors cannot hold short positions, i.e., $D_{ij} \geq 0$. While a complete inability to short is not realistic, it simplifies the analysis. In Section 7.1, I show that the qualitative nature of the results continues to hold when the supply of shortable shares is bounded as in Banerjee and Graveline (2013) and Nezafat and Schroder (2022). Throughout, I treat demand functions that differ only on a set of measure zero as equivalent and follow standard conventions on applying the law of large numbers with a continuum of random variables (e.g., Admati, 1985).

Climate risk disclosure. Prior to trade, each firm j discloses a public signal $\tilde{y}_j$ about its

[5]Specifically, this avoids having to deal with equilibria where measure zero sets of investors deviate from the behavior of other investors; this can be consistent with equilibrium because the behavior of such a set of investors has no impact on prices.

[6]In particular, see Pástor et al. (2021)'s analysis in their Section 5; their baseline model instead assumes that investors have different tastes for holding climate-exposed stocks.

climate beta. These signals may represent any information the firms have that is relevant to how their cash flows correlate with climate news. Firms' disclosures $\tilde{y}_j$ can be interpreted as signals of firms' exposures to either physical or transition risks, because the factor $\tilde{F}_C$ can be thought of broadly as capturing future regulations, demand shifts, and/or realized climate outcomes.

For example, the signals $\tilde{y}_j$ may be interpreted as firms' carbon disclosures, which are most clearly related to transition risk: firms with high emissions are likely to be subject to greater regulatory costs following negative climate news. Alternatively, the signals $\tilde{y}_j$ may be interpreted as information on the geography of firms' supply chains, which may be relevant to their physical risk exposures. For simplicity, I assume that firm j's signal $\tilde{y}_j$ does not provide information on other firms' climate betas, i.e., for $j_1 \neq j_2$, $\tilde{y}_{j_1} \perp \tilde{\beta}_{j_2}$. I let $\hat{\beta}_j \equiv \mathbb{E}\left[\tilde{\beta}_j|\tilde{y}_j\right]$ and re-index the firms so that investors' expectations of firm j's climate beta, $\hat{\beta}_j$, increase in j.[7]

4 Equilibrium

4.1 Initial Benchmarks

To clarify the drivers of the paper's main results, I first consider two benchmarks: one in which investors can trade in a climate derivative that is exclusively exposed to climate risk, and one in which investors can costlessly short sell. Throughout, I refer to an equilibrium with efficient risk sharing as one in which, after trade, the equilibrium is Pareto efficient in that investors' marginal utilities across states are proportional (i.e., the only difference in their marginal utilities stems from their average wealth). This is equivalent to investors having identical exposures to the systematic sources of risk in the model, $\tilde{F}_C$ and $\tilde{F}_M$, after trading. I let $EU_i = \mathbb{E}\left[u\left(\tilde{w}_i\right)\right]$ denote an investor of type $i \in \{G, B\}$'s expected utility.

Remark 1. ***Complete market benchmark.*** *Suppose that, in addition to the other assets, investors can trade in a derivative with payoff* $\tilde{x}_d = a_0 + a_C\tilde{F}_C$ *in zero net supply, where*

[7] Technically, when $\hat{\beta}_j$ has unbounded support, this implies that, upon re-indexing, there is no firm at one or both of the endpoints of $j \in [0, 1]$. Throughout, for notational brevity, I abuse notation when I refer to investors holding firms in regions that include one of these endpoints in the case in which $\hat{\beta}_j$ has support that is unbounded. For example, when I state that investors hold stocks in $[0, T]$, in the case in which $\hat{\beta}_j$ has support that is unbounded below, I simply refer to the investors as holding all firms with $\hat{\beta}_j \leq \hat{\beta}_T$.

$a_C \neq 0$. *Then, an investor of type $i \in \{G, B\}$'s equilibrium certainty equivalent satisfies:*

$$-\frac{1}{\rho}\ln\left(-\rho \times EU_i\right) =$$
$$\left(\kappa\mu_\beta + z_i\right)\left(\mu_C - \frac{\rho\sigma_C^2}{2}\left(\kappa\mu_\beta + \bar{z}\right)\right) + \kappa\left(\mu_M - \frac{\rho\kappa\sigma_M^2}{2}\right) + \frac{\rho\sigma_C^2}{2}\left(\kappa\mu_\beta + \bar{z}\right)\left(\bar{z} - z_i\right),$$

and firms' prices satisfy:

$$P_k = \hat{\beta}_k\left[\mu_C - \rho\left(\kappa\mu_\beta + \bar{z}\right)\sigma_C^2\right] + \mu_M - \rho\kappa\sigma_M^2.$$

In this equilibrium, risk sharing is efficient.

This result illustrates that a derivative whose payoff is solely driven by the climate would complete the market in the model. Thus, following arguments in line with the first welfare theorem, we obtain that risk sharing is efficient in equilibrium. Moreover, as is typical in factor-pricing models that take cash flows as the primitive, firms' prices are linear in their cash-flow betas (e.g., Hughes, Liu, and Liu, 2007). However, this benchmark is problematic in practice, as evidence suggests that effective climate derivatives are not available. Instead, investors appear to adjust their equity portfolios to hedge climate risk.

In their review of the climate finance literature, Giglio et al. (2021) argue, "... many of the effects of climate change are sufficiently far in the future that neither financial derivatives nor specialized insurance markets are available to directly hedge those long-horizon risks. Instead, investors are largely forced to insure against realizations of climate risk by building hedging portfolios on their own." On the practitioner side, Krueger et al. (2020) find that "... many market participants, including institutional investors, find climate risks difficult to price and hedge, possibly because of [...] challenges in finding suitable hedging instruments," but that many portfolio managers hedge climate risk by altering their equity holdings. For this reason, I do not allow investors to trade in a climate derivative moving forward.

I next consider the standard framework applied in disclosure theory in which investors are completely unconstrained in the portfolios they can hold, and in particular, do not face short-sales constraints. This assumption is common due to the tractability it provides. The next remark shows that, as long as investors' expectations of firms' climate exposures are not the same across firms, the equilibrium is equivalent to the complete market benchmark.

Remark 2. ***No short-sales constraints benchmark.*** *Suppose that the investors do not face short-sale constraints. Moreover, suppose that firms' expected climate betas are not homogenous, i.e., that there exist two disjoint intervals I_1, I_2 on which, $\forall k \in I_1, j \in I_2$,*

$\hat{\beta}_k \neq \hat{\beta}_j$. *Then, for* $i \in \{G, B\}$*, investors' demands satisfy:*

$$\textit{(i)} \int D_{ij} dj = \kappa; \tag{1}$$

$$\textit{(ii)} \int D_{ij} \hat{\beta}_j dj = \kappa \mu_\beta + \bar{z} - z_i. \tag{2}$$

Moreover, stock prices and the investors' expected utilities are identical to their values in the complete market benchmark.

This result states that, absent portfolio constraints, arbitrarily small differences in firms' expected climate betas are sufficient to enable investors to fully share arbitrarily large differences in climate risk. Such differences may result from a small amount of disclosure. Alternatively, they would also arise if one were to introduce a small amount of heterogeneity in investors' priors over these betas into the model. Thus, this result suggests that climate risk disclosure would not be essential barring portfolio constraints – but, in contrast to the Hirshleifer effect, also would not reduce investors' ability to share risk.[8]

Note that any demand functions that satisfy conditions (1) and (2) are consistent with an equilibrium (i.e., there are multiple equilibria where investors' demands across the stocks differ). This multiplicity is not a unique feature of the model, but instead arises in any model with an infinite number of firms. Intuitively, investors can create riskless stock portfolios that consist of offsetting long and short positions and add these to their equilibrium portfolios without changing their payoffs in any way.[9] As we will see, short-sales constraints help to discipline this behavior.

To see why the conditions on investors' demand functions stated in (1) and (2) equalize investors' climate exposures, note that, post trade, their exposures to the climate equal their prior exposure z_i plus the sum of their demand in each stock times the stock's expected climate beta, $\int D_{ij} \hat{\beta}_j dj$. For investor $i \in \{B, G\}$, given (2), this reduces to:

$$z_i + \int D_{ij} \hat{\beta}_j dj = \kappa \mu_\beta + \bar{z},$$

and so is identical across the investors. Moreover, condition (i) in the remark ensures that investors also share market risk equally, i.e., each investor's overall demand for equities, and thus their exposure to $\tilde{F}_M$, is the same.

The intuition for why risk sharing is always perfect in equilibrium is as follows. Green

[8]In contrast, public information concerning $\tilde{F}_C$ would reduce investor welfare via the Hirshleifer effect.

[9]This is similar to the observation that investor demands are not pinned down in models with both traded calls, puts, and the stock, given that the same payoff functions can be replicated by holding different portfolios of options.

investors purchase diversified portfolios that are long green firms and, in some cases, short brown firms, and brown investors hold the remainder of the market. When investors are highly uncertain about any given firm's climate exposure, the climate exposure of each dollar invested into such portfolios is minimal. However, as these portfolios are well diversified, if necessary, the investors can lever them until their climate risk exposures are fully shared. If the difference in the investors' climate exposures is large, or the difference across firms' expected climate betas is small, this requires a large amount of leverage and large short positions. This behavior does not appear common in practice, and the average short interest is typically around only 5% of shares outstanding (Beneish, Lee, and Nichols, 2015). Moreover, climate-based investment funds tend to be long only. This may be due to the plethora of risks and costs associated with shorting (e.g., Engelberg, Reed, and Ringgenberg, 2018) and motivates the focus on short-sale constraints in my main analysis.

4.2 Equilibrium with Short-Sales Constraints

I now consider the case in which investors cannot short and cannot trade in a climate derivative. In this case, the following condition determines when investors can use stocks to efficiently share risk.

Condition 1. The difference between the average expected climate betas of firms $j \in [\lambda_G, 1]$ and firms $j \in [0, \lambda_G]$ is sufficiently large relative to the difference in green and brown investors' endowments of climate risk:

$$\kappa \left[\frac{1}{\lambda_B} \int_{\lambda_G}^{1} \hat{\beta}_j dj - \frac{1}{\lambda_G} \int_{0}^{\lambda_G} \hat{\beta}_j dj \right] \geq z_G - z_B. \tag{3}$$

Observe that the left-hand side of (3) captures the difference in the average expected betas $\hat{\beta}_j$ among firms that have expected betas exceeding vs. falling short of $\hat{\beta}_{\lambda_G}$. This naturally tends to rise when the distribution of $\hat{\beta}_j$ is more dispersed. More informative disclosures create more dispersion in this distribution, i.e., they generate more variation in investors' posterior expectations $\hat{\beta}_j$, which increases the likelihood that this condition is satisfied. However, whether the condition is satisfied also depends on the amount of variation across firms' *true* climate betas. If equation (3) does not hold upon substituting firms' true climate betas β_j for their expected betas $\hat{\beta}_j$, then this equation will be violated even when risk disclosures are fully revealing. The right-hand side of (3) demonstrates that the amount of dispersion in firms' climate betas necessary to achieve efficiency depends on how much risk investors need to share to reach an efficient equilibrium, which is captured by $z_G - z_B$. In the next section, I analyze in more detail how the properties of the disclosure affect whether this condition holds.

In the next proposition, I show that Condition 1 is sufficient for the equilibrium with short-sale constraints to be Pareto efficient.

Proposition 1. *Suppose that Condition 1 holds. Then, in equilibrium, short-sale constraints do not bind. Investors' demands again satisfy equations (1) and (2). Stock prices and investors' expected utilities are equal to their values in the complete market benchmark, and the equilibrium is Pareto efficient.*

The intuition for this result is as follows. In an efficient equilibrium, green and brown investors must share market risk equally, which equates to each investor having an exposure of κ to the factor $\tilde{F}_M$. The strongest climate-hedging portfolio that green investors can construct that has such an exposure and clears the market holds $\frac{\kappa}{\lambda_G}$ shares of each of the stocks $[0, \lambda_G)$. Intuitively, including brown stocks would dilute the portfolio's climate exposure, and holding a more concentrated portfolio of green stocks would not clear the market, since there are too few outstanding shares to support such a portfolio.

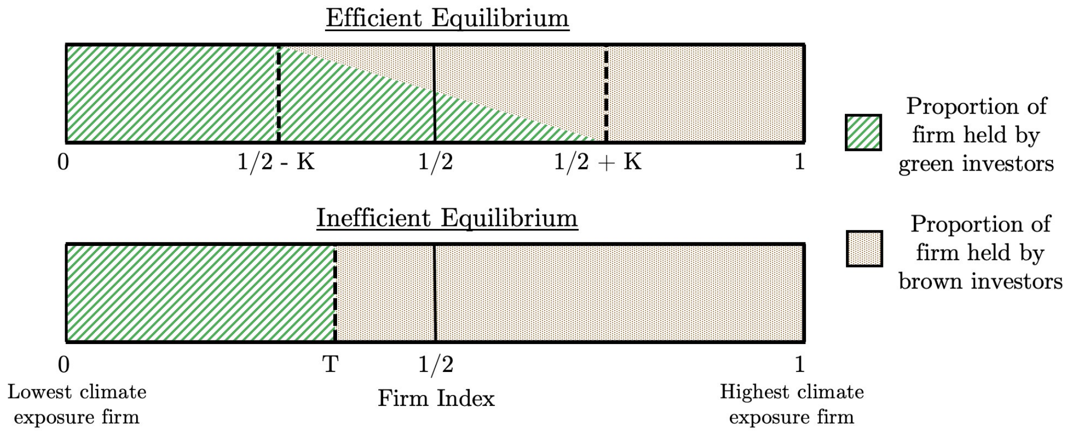
This implies that green investors hold all outstanding shares of stocks $[0, \lambda_G)$, and thus brown investors must hold all shares of stocks $(\lambda_G, 1]$. These holdings leave the green and brown investors with $z_G + \frac{\kappa}{\lambda_G} \int_0^{\lambda_G} \hat{\beta}_j dj$ and $z_B + \frac{\kappa}{\lambda_B} \int_{\lambda_B}^1 \hat{\beta}_j dj$ units of climate risk, respectively. Thus, inequality (3) asks whether these holdings leave green investors with less climate risk than brown investors. If they do, then there is an efficient equilibrium in which green investors hold a slightly less green portfolio than the one that holds only stocks $[0, \lambda_G)$, so that their exposures exactly equal those of brown investors. Equilibrium prices and demands then take the same form as in the case without short-sale constraints.

In contrast, if inequality (3) does not hold, the investors cannot construct long-only portfolios that simultaneously provide them with the efficient level of market and climate risk. I next characterize the equilibrium in this case, and verify that it is not Pareto efficient. To present the results in a parsimonious fashion, moving forward, I assume that $\hat{\beta}_j$ is continuously distributed with connected support.

Proposition 2. *Suppose that Condition 1 does not hold. Then, there exists a unique $T \in [0, 1)$ such that, in equilibrium, brown investors hold all shares of stocks $j > T$, taking positions of $\frac{\kappa}{\lambda_B}$ in each of these stocks, and green investors hold all shares of stocks $j < T$, taking positions of $\frac{\kappa}{\lambda_G}$ in each of these stocks. Green investors have a higher exposure to climate risk in equilibrium, i.e.,*

$$z_G + \frac{\kappa}{\lambda_G} \int_0^T \hat{\beta}_j dj - \left(z_B + \frac{\kappa}{\lambda_B} \int_T^1 \hat{\beta}_j dj \right) > 0.$$

Figure 1: Equilibrium Stock Allocations ($\lambda = \frac{1}{2}$)



Thus, the equilibrium is not Pareto efficient.

This proposition shows that the market is segmented, with green investors holding stocks with low expected climate betas and brown investors holding stocks with high expected climate betas. Green investors always hold more climate risk in equilibrium than brown investors, and so the equilibrium is not efficient. Intuitively, the equilibrium takes this threshold form because green investors continue to be exposed to a greater amount of climate risk than brown investors, and so they are willing to pay more for greener stocks.

Figure 1 contrasts this with an efficient equilibrium. In the figure, I assume that $\lambda_G = \frac{1}{2}$, which implies that, in an efficient equilibrium, the two groups of investors hold the same overall amount of the stock market. In an efficient equilibrium, the market is not fully segmented, as green investors do not need to hold exclusively the greenest firms to fully share their climate risk. Note the equilibrium in the efficient case is generally not unique, for the same reasons as in the case without short-sales constraints: investors can obtain the same risk exposures with different diversified portfolios. In contrast, the equilibrium in the inefficient case is always unique.

I next characterize basic properties of the inefficient equilibrium and discuss how investors' equilibrium holdings, as captured by T, are determined. To begin, I consider when the two investor groups participate in the market.

Corollary 1. *Brown investors always hold a positive measure of stocks. In equilibrium, green investors do not participate in the market (i.e., $T = 0$) when both:*

$$z_G > z_B + \frac{\kappa}{\lambda_B}\mu_\beta \quad \textit{and} \quad \left(\lim_{t\to 0}\hat{\beta}_t\right)\left(z_G - z_B - \frac{\kappa}{\lambda_B}\mu_\beta\right)\sigma_C^2 - \frac{\kappa\sigma_M^2}{\lambda_B} > 0. \tag{4}$$

Otherwise, green investors hold a positive measure of stocks (i.e., $T > 0$).

While brown investors always participate, green investors may abstain from participating from the market. However, green investors always participate when there are at least some stocks that are true climate hedges, i.e., when $\lim_{t\to 0}\hat{\beta}_t < 0$, since they are always willing to pay more for such hedges than are brown investors. As inequality (4) shows, green investors do not participate only when both all stocks are positively exposed to the climate *and* when the green investors have large climate exposures. Since, in practice, some firms in the economy (such as green technology firms) likely perform better following adverse than positive news on climate change, the model is consistent with the evidence that climate-conscious investors do participate in the market.

When both investor groups participate in equilibrium, they must be exactly indifferent between buying or selling a marginal share of stock T. As I show in the appendix, this implies that the equilibrium T must solve:

$$\hat{\beta}_T\sigma_C^2\left[\overbrace{z_G+\frac{\kappa}{\lambda_G}\int_0^T\hat{\beta}_j dj}^{\text{green investors' climate exposure}}-\overbrace{\left(z_B+\frac{\kappa}{\lambda_B}\int_T^1\hat{\beta}_j dj\right)}^{\text{brown investors' climate exposure}}\right]+\sigma_M^2\left[\overbrace{\frac{\kappa T}{\lambda_G}}^{\text{green investors' mkt. exposure}}-\overbrace{\frac{\kappa\,(1-T)}{\lambda_B}}^{\text{brown investors' mkt. exposure}}\right]=0. \quad (5)$$

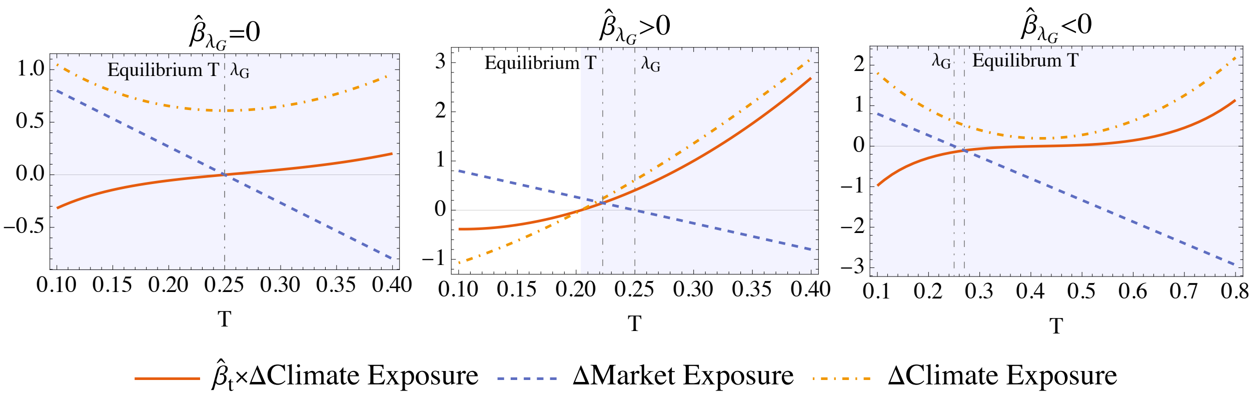
The idea is that, for the two investor groups to be indifferent between buying or selling a marginal share of stock T, the contribution of stock T to the risk of the two groups' portfolios must be identical. Since the two groups' climate and market risk exposures are not equal in an inefficient equilibrium, it must be that stock T has offsetting impacts on the relative climate and market risks of their portfolios. This leads to equilibrium condition (5), which states that the excess market risk faced by one group of investors precisely equals the excess climate risk of the other group scaled by stock T's climate beta. In general, this equation is non-monotonic in T and can have multiple zeroes, but the proof shows that only one solution satisfies the conditions for equilibrium.

Figure 2 illustrates how the equilibrium T is determined. In the plots, the green (brown) investors hold stocks $j \in [0, x)$ ($j \in (0, x]$). The yellow dot-dashed curves depict green investors' climate exposures less those of brown investors. The shaded regions indicate the areas over which green investors have higher climate exposures, which must hold at the equilibrium T. The solid red lines depict the contribution of stock x to the climate risk of green relative to brown investors, i.e., the first term in (5). The dashed blue lines depict -1 times the contribution of stock x to the market risk of green relative to brown investors, i.e., -1 times the second term in (5). Thus, the equilibrium T arises when these two curves intersect.

The figure separately depicts the cases in which the fraction of stocks that are true climate hedges (i.e., those with $\hat{\beta}_j < 0$) equals, falls short of, and exceeds the fraction of

Figure 2: Characterizing the Equilibrium Threshold T

This figure depicts how the equilibrium T is determined, showing how it balances the relative climate and market risks of the investor groups. In the figure, I assume that $\tilde{\beta}_j$ and $\tilde{y}_j$ are joint normal. In both plots, I set $\sigma_C = 2$, $\sigma_M = 1$, $z_G = 2$, $z_B = 1$, $\rho = 1$, $\lambda_G = 0.25$, $\kappa = 1$, and $\text{var}\left[\hat{\beta}_j\right] = 0.5$. In the left-hand, middle, and right-hand plots, I set $\mu_\beta = 0.337$, $\mu_\beta = 1$, and $\mu_\beta = 0.1$, respectively. The shaded regions indicate where green investors' climate exposures exceed those of brown investors, which is a necessary condition for equilibrium.



green investors λ_G, which correspond to $\hat{\beta}_{\lambda_G} = 0$, $\hat{\beta}_{\lambda_G} > 0$, and $\hat{\beta}_{\lambda_G} < 0$, respectively. While green investors always face more climate risk, this feature determines whether green investors hold more or less market risk than brown investors in equilibrium. The idea is as follows. Suppose that green investors hold the appropriate level of market risk, holding stocks $[0, \lambda_G]$. If this leaves them with less climate exposure than brown investors, then, following the reasoning given above, an efficient equilibrium would arise. Thus, if there is no efficient equilibrium, the green investors must remain over-exposed to climate risk. This implies that, if $\hat{\beta}_{\lambda_G} > 0$, brown investors are willing to pay strictly more for stocks $j \geq \lambda_G$ than are green investors, so that T must lie below λ_G in equilibrium. The opposite holds when $\hat{\beta}_{\lambda_G} < 0$. Finally, if $\hat{\beta}_{\lambda_G} = 0$, the equilibrium condition (5) is satisfied for $T = \lambda_G$. The next corollary formalizes this result.

Corollary 2. *Green investors are over-exposed (under-exposed) to market risk $\tilde{F}_M$ relative to an efficient equilibrium if and only if the measure of green investors in the market exceeds the measure of stocks that are expected to perform better following adverse climate shocks. Formally,*

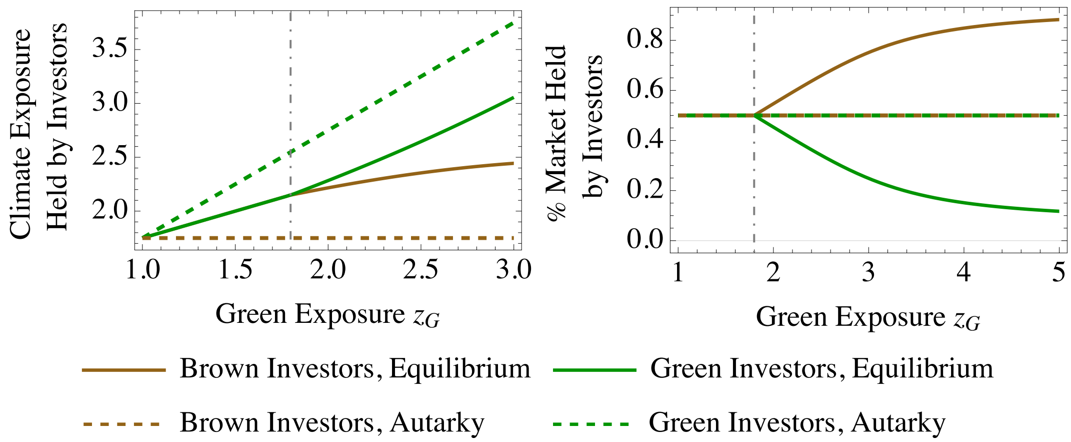
$$\hat{\beta}_{\lambda_G} > 0 \Rightarrow T < \lambda_G; \tag{6}$$

$$\hat{\beta}_{\lambda_G} < 0 \Rightarrow T > \lambda_G; \tag{7}$$

$$\hat{\beta}_{\lambda_G} = 0 \Rightarrow T = \lambda_G. \tag{8}$$

Figure 3: Investors' Exposures and Shareable Risk

This figure depicts green and brown investors' equilibrium exposures to climate and market risk as a function of the amount of climate risk that can be shared in equilibrium, as captured by z_G. In the figure, I assume that $\tilde{\beta}_j$ and $\tilde{y}_j$ are joint normal. In both plots, I set $\mu_\beta = 1.5$, $\sigma_C = 1$, $\sigma_M = 1$, $z_B = 1$, $\rho = 1$, $\lambda_G = 0.5$, $\kappa = 0.5$, and $\text{var}\left[\hat{\beta}_j\right] = 1$. The grey dashed lines demarcate the regions where risk sharing is vs. is not efficient in equilibrium. The solid lines indicate investors' equilibrium exposures, while the dashed lines indicate their exposures if they did not trade.



Note the case in which $\hat{\beta}_{\lambda_G} > 0$ is perhaps most empirically descriptive, as it is unlikely that a significant part of the market performs better following adverse climate news. While "pure-play" green stocks are likely to perform better given adverse climate shocks, such stocks appear to compose only a small part of overall market capitalization. Thus, the model is consistent with more climate-exposed investors investing less in the market (e.g., Ilhan, 2020).

Figure 3 depicts the investors' holdings of climate and market risk in equilibrium as a function of green investors' climate exposures z_G, which parameterizes the amount of risk that can be shared between the investor groups. The left-hand plot shows that green and brown investors' climate exposures grow in tandem until z_G crosses a cutoff. Beyond this point, the equilibrium is inefficient and green investors' climate exposures increase relative to those of brown investors. The right-hand plot similarly shows that green and brown investors' market exposures are equal for low values of z_G. However, since $\hat{\beta}_{\lambda_G} > 0$ in the example, for high values of z_G, green investors' market holdings decline: their desire to hedge their climate exposure distorts their equilibrium holdings of market risk.

It is possible that even without climate risk disclosure, investors' beliefs over firms' climate betas are highly dispersed, so that Condition 1 is satisfied and the equilibrium is efficient. I have ruled out this possibility by assuming investors start with common priors over firms' climate betas. However, recent empirical evidence suggests that, under current economic conditions, investors' beliefs over firms' climate betas are not highly dispersed. This evidence indicates that, even after accounting for the climate disclosures firm currently provide, investors cannot efficiently share climate risk. In particular, Engle, Giglio, Kelly, Lee, and Stroebel (2020) find that a dynamic equity portfolio optimized to hedge climate risk is at most 30% correlated with news on such risk, and Andersson, Bolton, and Samama (2016) show that a long-only portfolio optimized to reduce carbon emissions while maintaining a broad-market exposure eliminates only 50% of such emissions. The survey evidence in Krueger et al. (2020) and Ilhan et al. (2023) further suggests that institutional investors view existing climate risk disclosure as insufficient to enable them to efficiently hedge climate risk. For the same reason, I focus on inefficient equilibria in the next section.

5 Equilibrium Analysis

I next study the relationship between disclosure informativeness and efficiency, market valuations, and investor welfare.

5.1 Disclosure Informativeness and Efficiency

This section considers how the informativeness of firms' climate risk disclosures affects equilibrium risk-sharing efficiency. Formally, I define a disclosure policy to be more informative if it generates a mean-preserving spread in investors' posterior means. This notion is sometimes referred to as integral precision (Ganuza and Penalva, 2010), is implied by increases in informativeness in the Blackwell sense (Baker, 2006), and holds under several commonly-used parametric notions of informativeness. Moreover, it is clearly tied to Condition 1, which also concerns the distribution of investors' posterior means. The next proposition demonstrates how this notion relates to risk-sharing efficiency.[10]

Proposition 3. *Consider two disclosure policies associated with posterior expected climate betas $\hat{\beta}_j$ and $\hat{\beta}_j^{\dagger}$. The policy associated with $\hat{\beta}_j^{\dagger}$ is more informative than the policy associated*

[10]This result is not an immediate application of the definition of a mean-preserving spread, which concerns the integral of the CDF of a distribution. To see why, note that, letting $F_{\hat{\beta}}$ denote the CDF of $\hat{\beta}_j$:

$$\frac{1}{\lambda_B}\int_{\lambda_G}^{1}\hat{\beta}_j dj - \frac{1}{\lambda_G}\int_{0}^{\lambda_G}\hat{\beta}_j dj = \frac{1}{\lambda_B}\int_{\lambda_G}^{1}F_{\hat{\beta}}^{-1}(x)\,dx - \frac{1}{\lambda_G}\int_{0}^{\lambda_G}F_{\hat{\beta}}^{-1}(x)\,dx.$$

That is, the condition depends on the integral of the inverse CDF of $\hat{\beta}_j$, or equivalently, the conditional expectation of $\hat{\beta}_j$ given that it is truncated at $\hat{\beta}_{\lambda_G}$, rather than the CDF of $\hat{\beta}_j$.

with $\hat{\beta}_j$ if and only if, $\forall \lambda_G \in (0,1)$,

$$\frac{1}{\lambda_B}\int_{\lambda_G}^{1}\hat{\beta}_j^{\dagger}dj - \frac{1}{\lambda_G}\int_{0}^{\lambda_G}\hat{\beta}_j^{\dagger}dj > \frac{1}{\lambda_B}\int_{\lambda_G}^{1}\hat{\beta}_j dj - \frac{1}{\lambda_G}\int_{0}^{\lambda_G}\hat{\beta}_j dj,$$

i.e., the condition for the equilibrium to be efficient is strictly more likely to be satisfied.

This proposition shows that, under a more informative disclosure policy, risk sharing is more likely to be efficient in equilibrium. This is consistent with the intuition that, by creating more variation in investors' beliefs, a more informative policy helps investors to construct efficient hedging portfolios. Because an increase in the variance of a normal distribution generates a mean-preserving spread, an immediate consequence of this proposition is that, in the standard Gaussian setting, a more precise disclosure is more likely to result in efficient risk sharing. I formalize this in the next remark.

Remark 3. *Suppose that, $\forall j \in [0,1]$, $\tilde{\beta}_j \sim N(\mu_\beta, \sigma_0^2)$ and that the disclosed signals $\tilde{y}_j$ satisfy $\tilde{y}_j = \tilde{\beta}_j + \tilde{\varepsilon}_j$, where $\tilde{\varepsilon}_j \sim N(0, \sigma_\varepsilon^2)$ is independent of all other random variables. Then, there exists a threshold $v_E \in \mathcal{R}^+$ such that Condition 1 is satisfied if and only if:*

$$\sigma_{\hat{\beta}}^2 \equiv var\left[\hat{\beta}_j\right] = \frac{\sigma_0^4}{\sigma_0^2 + \sigma_\varepsilon^2} \geq v_E.$$

Therefore, if $\sigma_0^2 \geq v_E$, risk sharing is efficient in equilibrium if and only if the precision of the disclosure σ_ε^{-2} is sufficiently large.

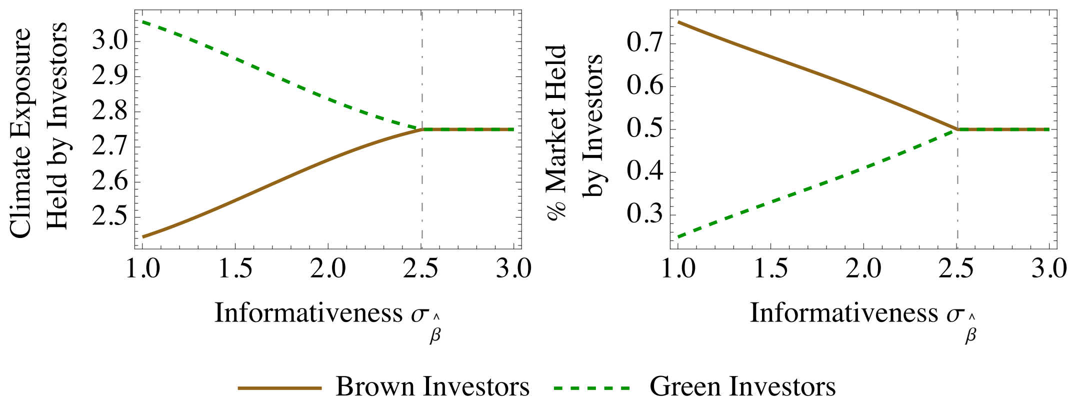
Figure 4 plots the relationship between disclosure informativeness and investors' equilibrium exposures to market and climate risk in the Gaussian setting. The plots focus on the case in which $\hat{\beta}_{\lambda_G} > 0$, so that green investors are inefficiently over-exposed to market risk in equilibrium. The figure illustrates that more informative climate disclosure raises (lowers) green (brown) investors' market holdings, and vice versa for investors' climate risk exposures, until they are equalized across the investor groups.

In the next corollary, I show that the model indeed predicts that more informative climate risk disclosure raises (lowers) the amount of market risk borne by green (brown) investors under two reasonable conditions. The first necessary condition is that green investors make up no more than half the market ($\lambda_G \leq \frac{1}{2}$), which is consistent with current empirical estimates.[11] The second condition is that the fraction of green investors is at least equal to the fraction of true climate hedges available in the market ($\hat{\beta}_{\lambda_G} > 0$), which, as previously discussed, appears empirically descriptive.

[11]See Pastor, Stambaugh, and Taylor (2023), who find that, "the total dollar ESG-related tilt is about 6% of the industry's AUM in equity investments in 2021."

Figure 4: Investors' Exposures and Disclosure Informativeness

This figure depicts green and brown investors' equilibrium exposures to climate and market risk. In the figure, I assume that $\tilde{\beta}_j$ and $\tilde{y}_j$ are joint normal. In both plots, I set $\mu_\beta = 1.5$, $\sigma_C = 1$, $\sigma_M = 1$, $z_G = 3$, $z_B = 1$, $\rho = 1$, $\lambda_G = 0.5$, and $\kappa = 0.5$. The grey dashed lines demarcate the regions where risk sharing is versus is not efficient in equilibrium.



Corollary 3. *Consider the Gaussian set up set forth in Remark 3, and suppose that $\hat{\beta}_{\lambda_G} > 0$ and $\lambda_G \leq \frac{1}{2}$. Then, an increase in $\sigma_{\hat{\beta}}$ increases (decreases) green (brown) investors' market holdings $\int D_{Gj}dj$ ($\int D_{Bj}dj$).*

One may also ask whether there are properties of a climate risk disclosure policy other than its overall informativeness that render it more useful in risk sharing (such as the relative informativeness of the policy for firms with high vs. low climate risk exposures). For the policy to increase risk-sharing efficiency *regardless of* λ_G, the "sufficiency" component of Proposition 3 implies that the policy must be strictly more informative. However, for specific values of λ_G, certain types of policies might be more effective in enabling risk sharing. To see why, note that, because $\mu_\beta = \int_0^{\lambda_G} \hat{\beta}_j dj + \int_{\lambda_G}^1 \hat{\beta}_j dj$, we can write:

$$\frac{1}{\lambda_B} \int_{\lambda_G}^1 \hat{\beta}_j dj - \frac{1}{\lambda_G} \int_0^{\lambda_G} \hat{\beta}_j dj = \frac{1}{\lambda_B} \mu_\beta - \left(\frac{1}{\lambda_B} + \frac{1}{\lambda_G} \right) \int_0^{\lambda_G} \hat{\beta}_j dj. \tag{9}$$

This shows that we can focus on investors' posterior expectations when they lie below λ_G, $\int_0^{\lambda_G} \hat{\beta}_j dj$, when determining whether a policy is more likely to lead to efficient risk sharing.

To see the implications of this, consider the case where the fraction of green investors λ_G is low. In this case, a disclosure policy need only be informative when a firm has a low climate beta to lead to an efficient equilibrium. Intuitively, if green investors can identify firms with low climate betas, brown investors can simply hold the rest of the market. Even though they do not know which of their individual holdings are the most climate exposed, their overall portfolio still provides them with the efficient level of climate risk. Conversely, we could also focus on investors' posterior expectations when they lie above λ_G, $\int_{\lambda_G}^{1} \hat{\beta}_j dj$. The idea is that disclosure on either sufficiently green or sufficiently brown firms is sufficient to achieve risk-sharing efficiency. I return to these points when studying voluntary disclosure.

5.2 Expected Valuations and Returns

As previously discussed, the firm's price in an efficient equilibrium is identical to the case without short-sales constraints. The next corollary characterizes firms' stock prices in the inefficient equilibrium.

Corollary 4. *Firms' prices are piecewise linear and continuous in $\hat{\beta}_k$, satisfying:*

$$P_k = \begin{cases} \hat{\beta}_k \left[\mu_C - \rho\left(\frac{\kappa}{\lambda_B}\int_T^1 \hat{\beta}_j dj + z_B\right)\sigma_C^2\right] + \mu_M - \rho\frac{\kappa}{\lambda_B}\kappa\left(1-T\right)\sigma_M^2 & \textit{for } k \geq T \\ \hat{\beta}_k \left[\mu_C - \rho\left(\frac{\kappa}{\lambda_G}\int_0^T \hat{\beta}_j dj + z_G\right)\sigma_C^2\right] + \mu_M - \rho\frac{\kappa}{\lambda_G}T\sigma_M^2 & \textit{for } k < T \end{cases}.$$

This result shows that prices are continuous, but have a different slope as a function of $\hat{\beta}_k$ depending on whether stock k is held by green or brown investors. Relative to stocks held by brown investors, stocks held by green investors exhibit a stronger relationship between firms' climate exposures and their valuations (and thus expected returns). This result follows from Corollary 2: in equilibrium, green investors remain more exposed to the climate than brown investors, and so, among the stocks they hold, there is a greater premium placed on climate exposures. This violates the classic linear factor pricing of arbitrage pricing theory, because arbitraging this relationship would require short selling.[12]

The upper-left-hand panel of Figure 5 depicts this pricing relationship and how it varies with disclosure informativeness. It shows that firms' prices are convex functions of their expected climate betas $\hat{\beta}_j$, but this convexity declines when risk disclosure is more informative, up until the equilibrium is efficient and prices are linear. When there is better climate risk disclosure, green investors hedge a larger portion of their climate exposures. Hence, they require a smaller premium to hold firms with higher climate betas, which reduces the slope of the relationship between firms' climate betas and their prices. The converse relationship holds among firms held by brown investors.

The upper-right-hand panel of this figure shows that climate risk disclosure alters the aggregate market price. Consistent with intuition, this price tends to be inflated relative to the case in which short-sales constraints do not bind (which occurs when $\sigma_{\hat{\beta}} \geq 2.5$). This

[12] The basic idea is analogous to Lintner (1969)'s findings in a setting with short-sales constraints but known risk exposures that "the equilibrium price of any i^{th} stock is independent of all the assessments and the risk aversion and the marginal real wealth certainty equivalents of all investors who do not hold that stock in general equilibrium."

follows directly from the upper-left-hand plot, which documents that price is convex in $\hat{\beta}_j$; thus, Jensen's inequality implies that variation in $\hat{\beta}_j$ raises the aggregate price. Moreover, the price is inverse-U shaped in $\sigma_{\hat{\beta}}$. Intuitively, for low values of $\sigma_{\hat{\beta}}$, there is little heterogeneity among stocks, and thus the Jensen's inequality effect is muted. Similarly, for high values of $\sigma_{\hat{\beta}}$, the equilibrium approaches an unconstrained equilibrium, i.e., the convexity in the price approaches zero, again reducing the Jensen's inequality effect.

Finally, the lower plot verifies that more informative disclosure leads firms with very low climate betas to have higher valuations. However, more informative disclosure can reduce the valuations of firms with moderately negative climate betas. Intuitively, more informative disclosure causes the firm's price to more accurately reflect its true beta, which increases the prices of firms with negative climate betas. However, it also affects the climate risk premium faced by such firms by lowering the climate risk exposures of green investors. In sum, the model suggests that, while climate risk disclosure increases risk-sharing efficiency, its impact on firms' overall market valuations, and the valuations of firms with moderately negative climate exposures, need not be positive. Future work may consider the implications of this effect on firms' production decisions.

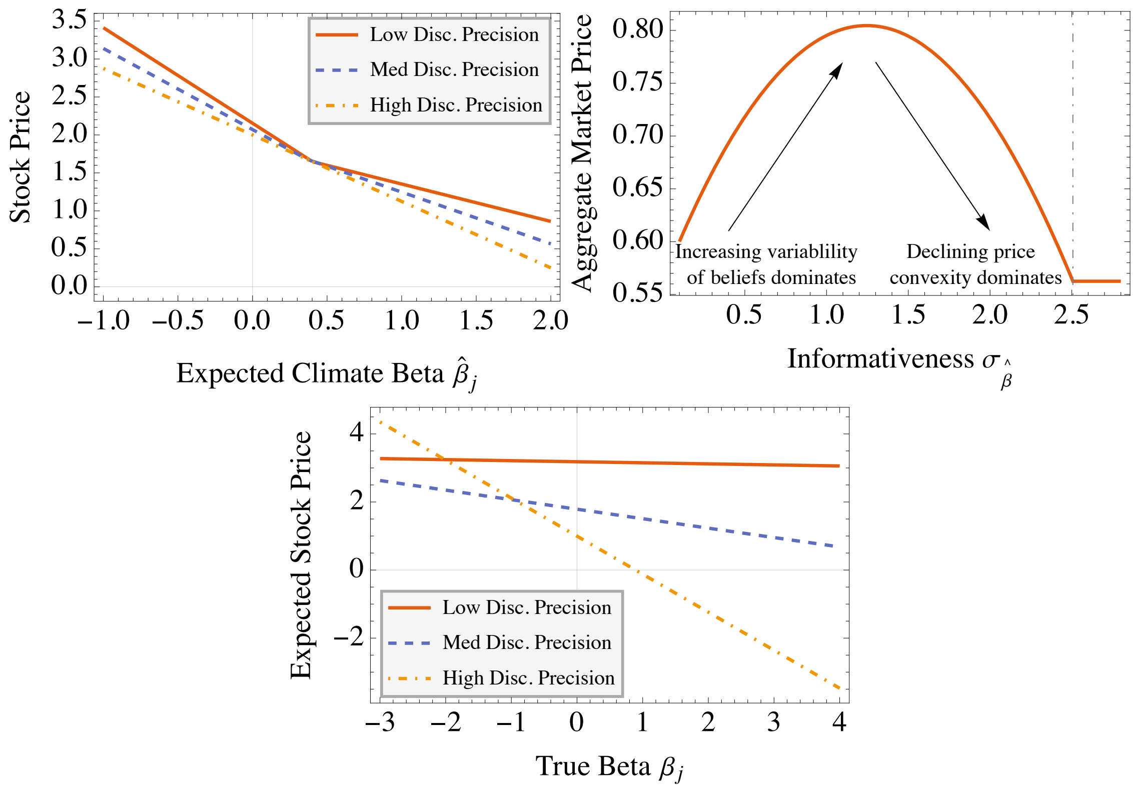
5.3 Investor Welfare and the Wealth Distribution

The results to this point address how climate risk disclosure impacts risk-sharing efficiency. However, investor welfare depends not only on risk-sharing efficiency, but also on the distribution of wealth across investors. For instance, higher-quality climate risk disclosure might ensure that investors' risk exposures are equalized, yet harm green investors by raising the price they must pay for the green stocks they purchase relative to the brown stocks they sell in equilibrium. Stated differently, because markets are incomplete, firms' disclosures can generate pecuniary externalities (Greenwald and Stiglitz, 1986). In such cases, a social planner would need to determine the relative weight they place on the expected utility of different agents to determine the optimal disclosure policy. Risk-sharing efficiency might be considered as more important than such distributional effects because, following the reasoning behind the second welfare theorem, redistribution via taxes or subsidies can address welfare differences driven by the wealth distribution. Nevertheless, it is useful to briefly consider how disclosure impacts the wealth distribution in addition to risk-sharing efficiency, as redistribution may be costly and difficult to achieve in practice.

To see formally how risk disclosure can alter the wealth distribution in my setting, note

Figure 5: Stock Prices, Climate Betas, and Disclosure Informativeness

This figure depicts individual stock prices and aggregate market prices. In the figure, I assume that $\tilde{\beta}_j$ and $\tilde{y}_j$ are joint normal. The upper-left-hand plot demonstrates how firms' stock prices vary with their expected climate betas, while the upper-right-hand plot demonstrates how the aggregate market price $\int_0^1 P_j dj$ varies with disclosure informativeness. The lower plot depicts the expected prices conditional on a firm's true climate beta, $\mathbb{E}[P_j|\beta_j]$, as a function of disclosure informativeness. In all plots, I set $\mu_\beta = 0.5$, $\mu_M = 2$, $\mu_C = -0.25$, $\sigma_C = 0.5$, $\sigma_M = 1$, $z_G = 4$, $z_B = 0$, $\rho = 1$, $\lambda_G = \frac{1}{2}$, and $\kappa = 1$. Low, medium, and high disclosure precision correspond to $\sigma_{\hat{\beta}} = 0.5$, $\sigma_{\hat{\beta}} = 1.5$, and $\sigma_{\hat{\beta}} = 3$, and the equilibrium is efficient only under high precision.



that investor $i \in \{G, B\}$'s certainty equivalent satisfies (see the appendix):

$$CE_i \equiv \begin{array}{c} \left(\int_0^1 D_{ij}\hat{\beta}_j dj + z_i\right)\mu_C + \left(\int_0^1 D_{ij} dj\right)\mu_M + \kappa\int_0^1 P_j dj - \int_0^1 D_{ij}P_j dj \\ -\frac{\rho}{2}\left(\int_0^1 D_{ij}\hat{\beta}_j dj + z_i\right)^2 \sigma_C^2 - \frac{\rho}{2}\left(\int_0^1 D_{ij} dj\right)^2 \sigma_M^2 \end{array}.$$

The term $\Pi_i \equiv \kappa\int_0^1 P_j dj - \int_0^1 D_{ij}P_j dj$ captures investors' costs of establishing their positions net of what they earn from selling their endowments in the market. As shown in the previous section, disclosure informativeness alters the relative prices of green and brown stocks, and thus the prices that the two investor groups pay to establish their portfolios. This transfers wealth from one group of investors to the other.

To see this more clearly, suppose that disclosure informativeness can be parameterized by a variable $\sigma_{\hat{\beta}}$ (as in, e.g., the case of normal distributions in the previous section). Then, we have:

$$\frac{d\Pi_G}{d\sigma_{\hat{\beta}}} = \overbrace{\frac{\partial \Pi_i}{\partial T}\frac{\partial T}{\partial \sigma_{\hat{\beta}}}}^{\text{cost of new positions}} + \underbrace{\frac{\partial \Pi_G}{\partial \sigma_{\hat{\beta}}}}_{\text{wealth transfer}}.$$

We see that a shift in $\sigma_{\hat{\beta}}$ has two effects on changes investors' net payments Π_i. First, as captured by the term $\frac{\partial \Pi_i}{\partial T}\frac{\partial T}{\partial \sigma_{\hat{\beta}}}$, it alters investors' equilibrium holdings (i.e., T), which reflects their optimal choice to adjust their portfolios and thus is not a wealth transfer.[13] Second, as captured by the term $\frac{\partial \Pi_G}{\partial \sigma_{\hat{\beta}}}$, it changes the prices at which investors execute their transactions, which captures how it shifts wealth across the investor groups.

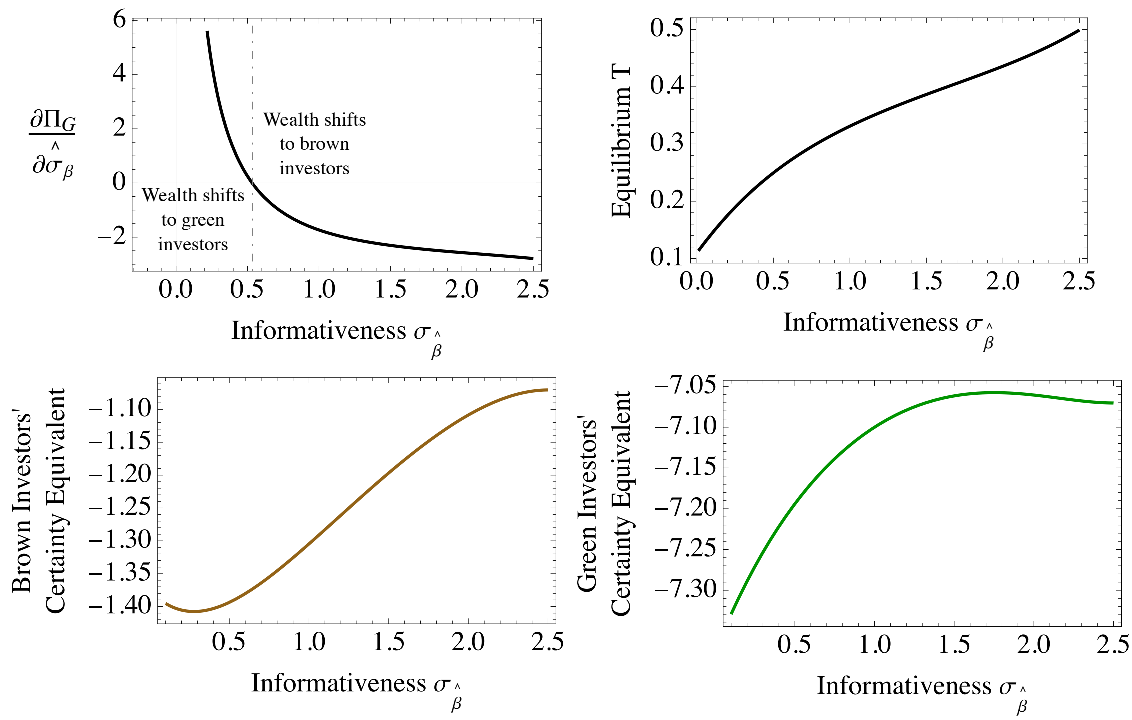
Figure 6 studies $\frac{\partial \Pi_i}{\partial \sigma_{\hat{\beta}}}$ in the normal-prior normal-signal case described in Remark 3. When risk disclosure is relatively uninformative (informative), a marginal increase in its informativeness transfers wealth from brown to green (green to brown) investors. This implies that for low (high) $\sigma_{\hat{\beta}}$, brown (green) investors' expected utility actually declines in $\sigma_{\hat{\beta}}$. Intuitively, suppose first that $\sigma_{\hat{\beta}}$ is low. The upper-right-hand panel shows that brown investors hold most of the market, purchasing shares from the green investors. Thus, brown investors are highly exposed to climate and market risk, which leads to large risk premia and enables them to establish their positions at a low price. Now, an increase in $\sigma_{\hat{\beta}}$ leads green investors to buy more of the market, which lowers brown investors' climate exposures and thus raises the risk premium, causing them to pay more to establish their positions. In contrast, for large values of $\sigma_{\hat{\beta}}$, green investors hold much more of the market, so that an analogous argument implies that the relative price of their portfolios increases in $\sigma_{\hat{\beta}}$.

It may be surprising that climate risk disclosure's impact on the amount that investors pay to establish their positions can dominate its positive impact on risk sharing and lead to a decline in welfare for one of the investor groups. To understand this, it is helpful to draw an analogy to prior work that studies the welfare-impact of disclosure on the level of firms' cash flows. This work finds that higher-quality disclosure transfers wealth from the purchasers of a security to the sellers of a security by lowering risk premia. As a result, higher-quality disclosure harms investors who tend to be net purchasers (e.g., Gao, 2010; Kurlat and Veldkamp, 2015; Dutta and Nezlobin, 2017). While these forces are more complex when considering risk disclosure and endogenous trade due to investor preferences, the reason they can lead to lower welfare for certain investors is similar.

[13] By "revealed preference," the net effect of this term and the remaining terms in investor welfare must be positive, or investors would not adjust their portfolios.

Figure 6: Investor Welfare and Disclosure Informativeness

This figure depicts wealth transfers across green and brown investors, and the impact on their welfare, as disclosure informativeness varies. In the figure, I assume that $\tilde{\beta}_j$ and $\tilde{y}_j$ are joint normal. In all plots, I set $\mu_\beta = \mu_M = 1$, $\mu_C = -0.5$, $\sigma_C = 1$, $z_G = 3$, $z_B = 1$, $\rho = 1$, and $\kappa = 0.5$.



6 Voluntary Disclosure and Disclosure Regulation

While the results to this point demonstrate that climate risk disclosure can enhance risk sharing, this alone does not imply that mandatory disclosure requirements are necessary to achieve efficient risk sharing. The reason is that firms are likely incentivized to voluntarily disclose information on their climate risks absent regulation, and empirical evidence suggests they indeed do so (Christensen, Hail, and Leuz, 2021). However, regulation may be warranted if firms' voluntary disclosures are not sufficient to lead to efficient risk sharing. Prior work has documented several frictions that limit firms' voluntary disclosures. The two most prominent of these frictions are costs to disclosure and investor uncertainty over whether managers are informed. I next explore firms' voluntary disclosure incentives and the role of regulation in the presence of these frictions.

Suppose now that each firm is run by a manager who potentially observes and can verifiably disclose the firm's climate beta.[14] The managers aim to maximize their firms'

[14] Assuming that managers observe their true climate beta is without loss of generality. If the managers observe noisy signals about their true climate beta, the results in this section continue to hold upon reinter-

share prices, taking as given the disclosure decisions of other managers in the economy. Characterizing the full set of disclosure equilibria in this setting is difficult when short-sales constraints bind and the equilibrium is not efficient. Instead, I focus on analyzing when there exists an efficient equilibrium.

I separately consider settings with disclosure costs and uncertainty over the managers' information endowments. Given that Proposition 1 holds under an essentially arbitrary distribution for $\hat{\beta}_j$, we can directly apply it in either case. Moreover, in an efficient equilibrium, firm j's price takes the same form as in Remark 1, adjusted downwards for any costs of disclosure. Suppose the manager of firm j discloses when $\tilde{\beta}_j \notin ND$ and does not disclose when $\tilde{\beta}_j \in ND$, and let $\hat{\beta}_{ND} \equiv \mathbb{E}\left[\tilde{\beta}_j | \tilde{\beta}_j \in ND\right]$.

6.1 Uncertainty over the Manager's Information Endowment

Consider first the Dye (1985) model of voluntary disclosure, where each firm's manager is informed with probability $p \in (0,1)$ and disclosure is costless. Then, from Proposition 1, disclosing is in the manager's best interest if and only if:

$$\beta_j \left[\mu_C - \rho\left(\kappa\mu_\beta + \bar{z}\right)\sigma_C^2\right] > \hat{\beta}_{ND}\left[\mu_C - \rho\left(\kappa\mu_\beta + \bar{z}\right)\sigma_C^2\right]$$
$$\Leftrightarrow \beta_j < \hat{\beta}_{ND},$$

where I applied the assumption that $\mu_C < 0$ and so $\mu_C - \rho\left(\kappa\mu_\beta + \bar{z}\right)\sigma_C^2 < 0$. This implies that only firms with low climate betas disclose, and so, as is typical in disclosure models, the equilibrium takes a threshold form (albeit, one where the manager discloses when her signal is *above* the threshold): for some $\mathcal{T} > \mu_\beta$, $ND = \{\beta_j : \beta_j > \mathcal{T}\}$.

This equilibrium "threshold disclosure policy" is well-suited towards achieving risk-sharing efficiency. To see why, recall from equation (9) that the condition for an efficient equilibrium to arise can be written as:

$$\frac{1}{\lambda_B}\mu_\beta - \left(\frac{1}{\lambda_B} + \frac{1}{\lambda_G}\right)\int_0^{\lambda_G} \hat{\beta}_j dj \geq \frac{z_G - z_B}{\kappa},$$

i.e., $\int_0^{\lambda_G} \hat{\beta}_j dj$ must be small relative to the prior expected beta, μ_β. Thus, it is sufficient that the disclosure policy is informative for firms with low climate betas $\tilde{\beta}_j$ (specifically, those below quantile λ_G of the distribution). The idea is that, as long as green investors can identify effective climate hedges, brown investors can simply hold the rest of the market. Even though the brown investors do not know which of their holdings are more exposed to

preting $\tilde{\beta}_j$ as firm j's expected beta conditional on the manager's noisy signal.

climate risk, they are certain of their overall allocation of climate risk.

In fact, the following proposition shows that voluntary disclosure is no less likely to lead to efficient risk sharing than full disclosure when: (i) green investors make up less than half the market, which is consistent with current empirical estimates (e.g., Pastor et al., 2023), and (ii) the distribution of $\tilde{\beta}_j$ is symmetric or positively skewed in that $\tilde{\beta}_{1/2} \leq \mu_\beta$.

Proposition 4. *Suppose that no more than $\frac{1}{2}$ of the investors are green ($\lambda_G \leq \frac{1}{2}$), that $\tilde{\beta}_{1/2} \leq \mu_\beta$, and that efficient risk sharing arises when the firm fully discloses conditional on being informed (i.e., Condition 1 is satisfied when the firm reveals its exposure given that it is informed). Then, there also exists an efficient equilibrium with voluntary climate risk disclosure.*

Intuitively, this proposition holds because, as is well known in the Dye (1985) model, firms with above-mean news (which corresponds to those with below-mean climate exposures) always disclose in equilibrium. Given a symmetric or positively-skewed distribution of climate exposures, this implies that firms with below-*median* climate exposures always disclose when they are informed. As I show in the proof, this implies that, for $\lambda_G \leq \frac{1}{2}$, we have $\int_0^{\lambda_G} \hat{\beta}_j dj = \int_0^{\lambda_G} \beta_j dj$. Thus, the condition for efficiency under voluntary disclosure is identical to that under full disclosure.

This proposition might suggest that disclosure mandates are not necessary if the Dye (1985) friction drives non-disclosure. However, it is important to note that certain forces outside the model may lead these mandates to raise risk-sharing efficiency. For instance, these mandates may force firms to produce new information on their climate exposures, such as information on their carbon emissions. Alternatively, standardized disclosure rules may render climate disclosures more credible and easier to parse than voluntary disclosures (Christensen et al., 2021). In addition, I next show that when disclosure is costly, disclosure mandates can play a more significant role.

6.2 Disclosure Costs

Following Jovanovic (1982) and Verrecchia (1983), suppose now that disclosing causes firms to incur a cost of $c > 0$. For instance, a firm may have soft information on its carbon emissions, but to credibly verify and disclose this information, it may need to pay fees to an auditor. Alternatively, by revealing information on its supply chain, a firm may enable its competitors to replicate parts of its strategy. For simplicity, suppose further that $\tilde{\beta}_j$ has a continuous, log-concave distribution with bounded support $\left[\underline{\beta}, \overline{\beta}\right]$, which ensures the equilibrium exists and is unique.[15]

[15] See Bagnoli and Bergstrom (2005); similar results hold when $\tilde{\beta}_j$ is normally distributed.

In this case, firm j's manager finds it optimal to disclose if and only if:

$$\left[\mu_C - \rho\left(\kappa\mu_\beta + \bar{z}\right)\sigma_C^2\right]\left(\beta_j - \hat{\beta}_{ND}\right) > c \Leftrightarrow \beta_j < \hat{\beta}_{ND} + \frac{c}{\mu_C - \rho\left(\kappa\mu_\beta + \bar{z}\right)\sigma_C^2}.$$

Again, in equilibrium, only firms with low climate betas disclose, and in particular:

$$ND = \left[\underline{\beta}, \hat{\beta}_{ND} + c\left[\mu_C - \rho\left(\kappa\mu_\beta + \bar{z}\right)\sigma_C^2\right]^{-1}\right].$$

As disclosure costs grow large, voluntary disclosure becomes completely uninformative, and so, in any equilibrium, $\frac{1}{\lambda_B}\int_{\lambda_G}^{1}\hat{\beta}_j dj - \frac{1}{\lambda_G}\int_0^{\lambda_G}\hat{\beta}_j dj \to 0$, i.e., an efficient equilibrium does not exist. In contrast, as $c \to 0$, in any equilibrium, the manager almost always discloses $\tilde{\beta}_j$. Hence, if there exists an efficient equilibrium under full disclosure, there also exists an efficient equilibrium under voluntary disclosure when disclosure costs are low.

Proposition 5. *Suppose that full disclosure leads to efficient risk sharing, i.e., Condition 1 is satisfied when $\tilde{y}_j = \tilde{\beta}_j$. Then, there also exists an efficient equilibrium with voluntary climate risk disclosure if and only if the disclosure cost c is sufficiently low.*

This suggests that disclosure mandates may be necessary to achieve risk-sharing efficiency when climate disclosure is sufficiently costly. Importantly, however, the welfare impact of such mandates also depends upon how disclosure costs enter the welfare function. These costs reduce welfare if they are deadweight losses, e.g., resources invested in verifying firms' carbon emissions. In this case, regulation is only useful when risk-sharing benefits, together with any other benefits not considered in the model, outweigh the direct costs of disclosure. In contrast, disclosure costs need not reduce welfare if they are proprietary. For instance, revealing the details of a firm's supply chain may harm its profitability by aiding its competitors, but, netting across firms, have little, or even a positive, overall impact on welfare. In this case, a disclosure mandate is more likely to increase welfare.

7 Additional Analyses

In this section, I demonstrate that the results are robust to allowing for limited short selling, investors to have different preferences for climate risk rather than different exposures, and introducing index investors who lead imperfect market segmentation.

7.1 Allowing for Constrained Short Selling

I next demonstrate that my results are robust to relaxing the assumption that investors are completely unable to short. Specifically, I now allow investors to short, but only up to a finite multiplier ξ of the shares outstanding of any given stock (Banerjee and Graveline, 2013). This specification has several advantages: it retains tractability, leads to some degree of short selling in equilibrium, consistent with the non-zero but positive short-selling observed in practice (Beneish et al., 2015), and accurately reflects share-borrowing constraints in practice. For example, shares held in non-margin accounts typically cannot be lent out, and various institutions including ETFs are constrained to lending out at most a fraction of their shares. The next proposition re-derives the main results under this alternative constraint.[16,17]

Proposition 6. *There exists an efficient equilibrium if and only if:*

$$\left(\frac{\kappa\xi}{\lambda_G}+\frac{\kappa\left(1+\xi\right)}{\lambda_B}\right)\int_{\frac{\xi+\lambda_G}{2\xi+1}}^{1}\hat{\beta}_j dj-\left(\frac{\kappa\left(1+\xi\right)}{\lambda_G}+\frac{\kappa\xi}{\lambda_B}\right)\int_{0}^{\frac{\xi+\lambda_G}{2\xi+1}}\hat{\beta}_j dj\geq z_G-z_B. \tag{10}$$

Moreover,

(i) This condition is strictly more likely to be satisfied as ξ increases.

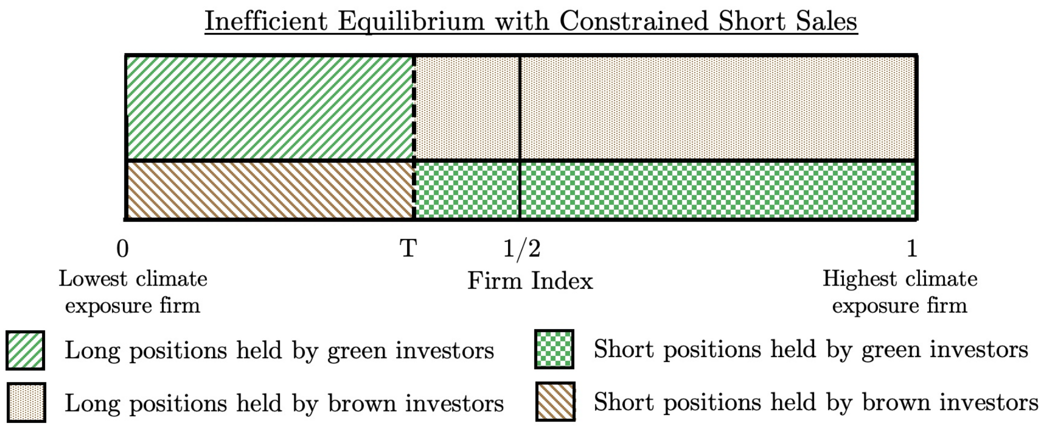
(ii) In an inefficient equilibrium, there exists a $T\in[0,1)$ such that green investors take long positions of $\frac{\kappa(1+\xi)}{\lambda_G}$ in stocks $[0,T)$ and short positions of $\frac{\kappa\xi}{\lambda_G}$ in stocks $(T,1]$, and brown investors take long positions of $\frac{\kappa(1+\xi)}{\lambda_B}$ in stocks $(T,1]$ and short positions of $\frac{\kappa\xi}{\lambda_B}$ in stocks $[0,T)$.

This proposition shows that the condition for an efficient equilibrium takes a similar form to the baseline case, capturing the difference between the expected climate betas of firms with high vs. low values of these betas. However, the ability to short relaxes this condition: green investors can short brown stocks and brown investors can short green stocks, and this generates a greater supply of shares for the investors groups to long. While this helps to equalize their exposures, when ξ is not too large, climate risk disclosure is still necessary to ensure that the equilibrium is efficient. When the equilibrium is inefficient, it again takes

[16] Similar results to those in this section would also hold if only a fraction of investors, such as hedge funds, could take (bounded) short positions. This would create a greater supply of highly-climate exposed shares for constrained green and brown investors to purchase. However, it need not lead to efficiency absent sufficient disclosure.

[17] One may also ask whether short-selling costs would be a viable alternative to limit short-selling in the model. There are two concerns with this approach. First, to understand disclosure's impact on welfare, one would need to consider who these costs accrue to. Second, without a cap on short-selling, short-selling costs generate a technical problem in the model. In particular, given such costs, investors would seek to trade very large amounts in the stocks with the largest absolute expected climate betas, as this would give them the greatest climate exposure at the least cost. Given a continuous distribution of $\hat{\beta}_j$, this leads investors to face an ill-defined optimization problem.

Figure 7: Equilibrium Stock Allocations with Limited Shorting ($\lambda = \frac{1}{2}$)



a threshold form, as illustrated in Figure 7. Green investors short stocks with high climate exposures to hedge their outside exposures, while brown investors short stocks with low exposures to take advantage of their inflated prices.

7.2 Direct Preferences for Climate Exposure

The analysis to this point focuses on gains from trade that result from investors sharing their non-tradable risk exposures using stocks. However, much of the trade in climate-related funds appears to be motivated by preferences for holding certain types of stocks, and this has been the focus of study in recent work (Friedman and Heinle, 2016; Goldstein et al., 2022; Chaigneau and Sahuguet, 2023). This raises the question of whether the results would also apply when investors are driven to trade climate-exposed stocks by their preferences rather than exposures. At an intuitive level, the mechanism underlying the main results – that climate risk disclosure helps investors to identify climate-exposed stocks – should extend to this alternative case. In this section, I verify that this holds.

Suppose now that investors have the same exposures to the climate ($z_G = z_B$), but that green (brown) investors obtain direct certainty-equivalent impact of $b_G\tilde{\beta}_j$ ($b_B\tilde{\beta}_j$) from holding stock j, where $b_G < b_B$, which captures a direct preference for climate-exposed stocks.[18] This specification, in which investors' preferences are related to firms' climate exposures, follows Section 3 in Pástor et al. (2021). To provide an example of how this

[18] The impact of disclosure on welfare in this case is identical to the case in which investors' private benefits from holding a stock increase in their expectation of its climate beta rather than the true climate beta. The reason is that investors hold diversified portfolios and climate betas are idiosyncratic, and so their expectation of their portfolio's beta aligns with its true climate beta. However, under this alternative, since investors' preferences depend not on the state of the world but on their beliefs, it is more difficult to define risk-sharing efficiency.

may arise, suppose that green investors have preferences for holding stocks with positive externalities. The same stocks that generate such externalities, such as green technology firms, are plausibly less exposed to transition risk because they may benefit from regulatory shifts or changes in consumer preferences following adverse climate news. A natural special case of this specification is when brown investors do not care about firms' climate exposures, while green investors prefer to hold firms with lower exposures; this corresponds to $b_G < 0$ and $b_B = 0$.

An alternative, mathematically-equivalent interpretation of this specification is that investors "agree to disagree" about how firms' climate exposures impact their *expected* cash flows. Under this interpretation, $b_G - b_B$ captures the difference in how green and brown investors expect a unit increase in climate exposure to impact a firm's expected cash flows. Note, however, that in this case, we must take their expected utilities under their subjective beliefs as the object of interest.

Proposition 7. *Suppose that climate exposures directly impact investors' certainty equivalents. Short-sale constraints do not bind and investors' marginal utilities across states are equalized in equilibrium if and only if:*

$$\frac{1}{\lambda_B}\int_{\lambda_G}^{1}\hat{\beta}_j dj - \frac{1}{\lambda_G}\int_{0}^{\lambda_G}\hat{\beta}_j dj \geq \frac{b_B - b_G}{\rho\kappa\sigma_C^2}.$$

This proposition shows that the results with outside exposures and direct preferences are close to identical. The only difference between is that, unlike investors' outside exposures, investors' private benefits are scaled by risk aversion ρ times the uncertainty over climate risk σ_C^2 in the condition for an efficient equilibrium to arise. The follows because investors optimal holdings of climate risk trade off their private benefits and risks from holding climate-exposed stocks. In contrast, in the private exposures case, there is no such trade-off because increases in climate risk also increase investors' demand for climate hedges.

7.3 Index Investors, Segmentation, and Risk-Sharing Efficiency

In the inefficient equilibrium, which, as previously argued, is more likely to be descriptive of existing market conditions, the market is fully segmented, with either green or brown firms holding the entirety of certain firms. In reality, however, firms' shareholder bases are diverse, including investors who do not appear to invest primarily based on climate exposures. One key reason that we do not see segmentation in the market is that investors often invest via broad-market, value-weighted index funds, which do not adjust their holdings based on firms' prices nor their climate exposures. Thus, a natural way to test the robustness of the findings to incomplete market segmentation is to introduce such indexers into the model.

I introduce indexers by assuming a fraction of investors must hold an equal amount of the outstanding shares of each stock, which reflects broad-market, value-weighted indexing (as in Coles, Heath, and Ringgenberg, 2022). These investors can be thought of as seeking to minimize transactions costs or lacking the knowledge necessary to trade based on prices and climate exposures. Formally, assume that a fraction of investors $\psi \in [0, 1)$ are indexers. Green and brown investors make up the same fraction of the remaining market participants as in the benchmark case, i.e., a fraction $(1 - \psi)\,\lambda_G$ of investors are green and a fraction $(1 - \psi)\,\lambda_B$ of investors are brown. Similar to the green and brown investors, I assume that indexers are endowed with κ shares of each stock and do not use leverage, which implies that they continue to hold κ shares of each stock in equilibrium. Importantly, maintaining a unit mass of investors ensures that, as ψ varies, the risk-bearing capacity of the market stays fixed.

I next show that the fraction of investors who are indexers has no effect on the equilibrium that plays out between the green and brown investors. Notably, the share holdings of each green and brown investor are the same with indexers ($\psi > 0$) and without indexers ($\psi = 0$), as are the conditions for an efficient equilibrium. This verifies that the results do not depend on a fully-segmented market.

Proposition 8. *The fraction of investors in the market who are indexers ψ does not affect the condition that determines whether the equilibrium is efficient, and it does not affect green and brown investors' equilibrium portfolios nor the price of each stock.*

Intuitively, indexers do not systematically alter the composition of stocks in the economy. As the fraction of indexers increases, they hold more of the firms with the lowest and highest climate exposures, which leaves fewer shares of such stocks for green and brown investors to purchase. However, when there are more indexers, the fraction of green and brown investors also declines, and so the total amount of risk to be shared in equilibrium declines. These two effects precisely offset.[19] Note this proposition implies that increases in disclosure informativeness will have similar welfare effects to those in the baseline model. It implies that disclosure affects green and brown investors' holdings and the prices they pay for these holdings in an identical manner as in the baseline model. Moreover, the welfare of the indexers does not depend on disclosure because they simply hold the shares they are endowed with.

[19] An increase in indexing by investors who would otherwise abstain from participating in the market would render efficient risk sharing between the remaining investors more difficult to achieve. This would effectively reduce the supply of shares κ to be traded by the remaining investors, which can makes the efficiency condition less likely to hold.

8 Empirical Implications

The paper's main results concern welfare and risk-sharing efficiency, which are challenging constructs to directly measure. However, the model generates several predictions that can be used to test whether the forces at play are empirically relevant. Moreover, these predictions are relevant to existing work on pricing and trading of climate risk, and can be tested using the measures recently developed for assessing climate exposures based on public disclosures or covariances in returns (e.g., Pástor, Stambaugh, and Taylor, 2022; Sautner, Van Lent, Vilkov, and Zhang, 2023). I focus on the model's predictions under an inefficient equilibrium. The reason is that, as previously discussed, existing empirical and survey evidence suggests that investors hedge climate risk in equities, but cannot fully and efficiently do so (Engle et al., 2020; Krueger et al., 2020; Ilhan et al., 2023; Pastor et al., 2023).

Pricing of climate risk and climate disclosure. Several recent studies analyze the relationship between firms' climate exposures and their expected returns, often treating climate-related risk as a priced risk factor (e.g., Chava, 2014; Bolton and Kacperczyk, 2021; Pedersen et al., 2021; Pástor et al., 2022). Corollary 4 predicts that the extent to which a firm's climate risk exposure is associated with its expected returns depends upon the firm's investor base. This interaction implies that a single risk factor cannot account for the pricing of climate risk. Notably, the model predicts that the climate risk premium is smaller (i.e., greenness is less negatively related to expected returns) among firms held primarily by brown investors than among firms held primarily by green investors because, given short-sales constraints, stock prices reflect the preferences of the investors who hold them in equilibrium.

Corollary 4 further predicts that economy-wide shifts in climate risk disclosure alter the pricing of climate risk. When such disclosure becomes more informative, green investors typically better hedge their climate risk exposure in equilibrium, which reduces the climate risk premium among stocks held by green investors, but increases it among those held by brown investors. Finally, this corollary predicts that, in cross-sectional analyses, firms that disclose higher climate exposures (e.g., carbon emissions) should have higher expected future returns, and this relationship should be concave, decreasing in slope as firms' investor bases shift towards brown investors.

Climate risk disclosure, trade, and investor holdings. Corollary 3 predicts that economy-wide increases in climate disclosure (such as those created by disclosure regulation) increase the willingness of green investors to hold equities. Intuitively, by default, green investors are dissuaded from heavily participating in the market because the typical stock is positively exposed to the climate. However, climate disclosure enables them to identify and purchase the green segment of the stock market. This finding is relevant to empirical work linking investors' non-tradable climate exposures to their equity holdings (Ilhan, 2020). The analysis in Section 7.1 further suggests that climate risk disclosure may reduce the amount of short-selling in climate-exposed stocks in equilibrium. The reason is that climate risk disclosure eliminates the need to short sell to efficiently share climate risk.[20]

Voluntary climate risk disclosure. The results in Section 6 on voluntary climate risk disclosure are consistent with existing evidence and offer some new predictions. In equilibrium, because firms with lower climate exposures are more likely to disclose, the model predicts that the amount of climate disclosure a firm provides to the market is positively associated with investment by climate-conscious investors. The evidence in Ilhan et al. (2023) corroborates this finding using various metrics of voluntary climate disclosure. The model further predicts that firms that disclose information on their climate exposures should have future returns that covary less with future adverse climate news than firms that do not disclose such information. As such, disclosing firms earn lower expected future returns than firms that do not disclose.

9 Conclusion

This paper studies the impact that climate risk disclosure has on investors' ability to share climate risk in the financial market. I find that, in an unconstrained economy, such disclosure is not necessary for efficient risk sharing. However, upon accounting for investors' short-sale constraints, climate risk disclosure can be essential to enable investors to form efficient climate-hedging portfolios. The intuition is simple: such disclosure enables investors to identify firms with the most positive and negative climate exposures, and adjust their portfolios accordingly. However, climate risk disclosure can transfer wealth across green and brown investors, and in some cases, can lower one of these group's welfare. When short-sale constraints bind, prices are decreasing but convex functions of firms' climate risk exposures, and the extent of this convexity declines with the informativeness of firms' climate risk disclosures. Finally, firms' equilibrium voluntary disclosure behavior is effective at enabling efficient risk sharing except when disclosure is highly costly or the distribution of climate betas is highly negatively skewed. This yields conditions on when disclosure mandates can be effective.

My model offers a tractable means to study short-sales constraints with heterogenous firms and investors. A similar modeling approach could be applied to study questions related

[20] Technically, when the condition for an efficient equilibrium to arise without short selling is satisfied, there exist equilibria in which investors may still short sell even though they do not need to do so. However, introducing small frictions to shorting would eliminate such equilibria.

to sorting among investors and firms that extend beyond the scope of climate risk. Moreover, the model could be extended along several dimensions. One avenue would be to explore the implications of the results that I document on firm pricing for firms' incentives to invest in green vs. brown projects. This would enable the model to speak to the real effects of climate risk disclosure. The result that moderately green firms can receive lower valuations when firms' climate disclosures are more precise suggests that the relationship between production externalities and disclosure would not be obvious in such a setting. Alternatively, one could consider separately allowing for climate-focused long-short funds and long-only funds, or explicitly model the securities lending market.

A Proofs

Proof of Remark 1

Let D_{id} denote investor $i \in \{G, B\}$'s demand for the derivative and let P_d denote the derivative's price. Let EU_i denote investor i's ex-ante expected utility given a demand function $\left\{D_{id}, \{D_{ij}\}_{j\in[0,1]}\right\}$. This reduces as follows:

$$
\begin{aligned}
EU_i &\equiv -\frac{1}{\rho}\mathbb{E}_i\left[\exp\left(-\rho D_{id}\left(\tilde{x}_d - P_d\right) - \rho\int D_{ij}\left(\tilde{\alpha}_j + \tilde{\beta}_j\tilde{F}_C + \tilde{F}_M - P_j\right)dj - \rho z_i\tilde{F}_C - \rho\kappa\int P_j dj\right)\right] \\
&= -\frac{1}{\rho}\mathbb{E}_i\left[\exp\left(-\rho D_{id}\left(\tilde{x}_d - P_d\right) - \rho\int D_{ij}\left(\tilde{\beta}_j\tilde{F}_C + \tilde{F}_M - P_j\right)dj - \rho z_i\tilde{F}_C - \rho\kappa\int P_j dj\right)\right] \\
&= -\frac{1}{\rho}\mathbb{E}_i\left\{\mathbb{E}_i\left[\exp\left(\begin{array}{c} -\rho D_{id}\left(\tilde{x}_d - P_d\right) - \rho\int D_{ij}\left(\tilde{\beta}_j\tilde{F}_C + \tilde{F}_M - P_j\right)dj \\ -\rho z_i\tilde{F}_C - \rho\kappa\int P_j dj \end{array}\right) | \left\{\tilde{\beta}_j\right\}_{j\in[0,1]}\right]\right\} \\
&= -\frac{1}{\rho}\mathbb{E}_i\left[\exp\left(\begin{array}{c} -\rho\left(a_C D_{id} + \int D_{ij}\tilde{\beta}_j dj + z_i\right)\mu_C + \frac{\rho^2}{2}\left(a_C D_{id} + \int D_{ij}\tilde{\beta}_j dj + z_i\right)^2\sigma_C^2 \\ -\rho\left(\int D_{ij}dj\right)\mu_M + \frac{\rho^2}{2}\left(\int D_{ij}dj\right)^2\sigma_M^2 + \rho\int D_{ij}P_j dj \\ +\rho D_{id}\left(P_d - a_0\right) - \rho\kappa\int P_j dj \end{array}\right)\right].
\end{aligned}
$$

I will conjecture and verify an equilibrium in which the variance of $D_{ij}\tilde{\beta}_j$ is uniformly bounded from above, as a function of j. This allows us to apply Markov's strong law of large numbers to obtain:[21]

$$
\int D_{ij}\tilde{\beta}_j dj = \int \mathbb{E}\left[D_{ij}\tilde{\beta}_j\right]dj.
$$

Now, since D_{ij} is measurable with respect to $\hat{\beta}_j = \mathbb{E}\left[\tilde{\beta}_j|\tilde{y}_j\right]$, we obtain:

$$
\begin{aligned}
\mathbb{E}\left[D_{ij}\tilde{\beta}_j\right] &= \mathbb{E}\left\{\mathbb{E}\left[D_{ij}\tilde{\beta}_j|\hat{\beta}_j\right]\right\} \\
&= \mathbb{E}\left\{D_{ij}\mathbb{E}\left[\tilde{\beta}_j|\hat{\beta}_j\right]\right\} \\
&= \mathbb{E}\left[D_{ij}\hat{\beta}_j\right].
\end{aligned}
$$

Thus, $\int D_{ij}\tilde{\beta}_j dj = \int D_{ij}\hat{\beta}_j dj$; substituting this into (14), we obtain:

$$
EU_i = \exp\left(\begin{array}{c} -\rho\left(a_C D_{id} + \int D_{ij}\hat{\beta}_j dj + z_i\right)\mu_C + \frac{\rho^2}{2}\left(a_C D_{id} + \int D_{ij}\hat{\beta}_j dj + z_i\right)^2\sigma_C^2 \\ -\rho\left(\int D_{ij}dj\right)\mu_M + \frac{\rho^2}{2}\left(\int D_{ij}dj\right)^2\sigma_M^2 + \rho\int D_{ij}P_j dj + \rho D_{id}\left(P_d - a_0\right) - \rho\kappa\int P_j dj \end{array}\right).
$$

[21] See Theorem D.8 in Greene (2012). Following standard conventions, I rely on a continuum version of this theorem; see Uhlig (1996) for the necessary technical conditions for this to apply.

Let Ξ denote the expression in this exponential. Now, observe that:

$$\begin{aligned}
&\frac{\partial EU_i}{\partial D_{ik}} \\
&= \frac{\partial}{\partial D_{ik}} \mathbb{E}\left[-\exp\left(\Xi\right)\right] \\
&= \left(\rho\hat{\beta}_k\mu_C - \rho^2\hat{\beta}_k\left(a_C D_{id} + \int D_{ij}\hat{\beta}_j dj + z_i\right)\sigma_C^2 + \rho\mu_M - \rho^2\left(\int D_{ij}dj\right)\sigma_M^2 - \rho P_k\right)\exp\left(\Xi\right).
\end{aligned} \tag{11}$$

Moreover,

$$\frac{\partial EU_i}{\partial D_{id}} = \left(\rho a_C\mu_C - \rho^2 a_C\left(a_C D_{id} + \int D_{ij}\hat{\beta}_j dj + z_i\right)\sigma_C^2 - \rho\left(P_d - a_0\right)\right)\exp\left(\Xi\right).$$

Setting this equal to zero yields that:

$$\begin{aligned}
&\left(\rho a_C\mu_C - \rho^2 a_C\left(a_C D_{id} + \int D_{ij}\hat{\beta}_j dj + z_i\right)\sigma_C^2 - \rho\left(P_d - a_0\right)\right)\exp\left(\Xi\right) = 0 \\
&\qquad\Leftrightarrow -\rho\left(a_C D_{id} + \int D_{ij}\hat{\beta}_j dj + z_i\right)\sigma_C^2 = \frac{P_d - a_0 - a_C\mu_C}{a_C}.
\end{aligned} \tag{12}$$

Substituting this final expression into the first-order condition for stock k, $\frac{\partial EU_i}{\partial D_{ik}} = 0$, from (11), we obtain:

$$\begin{aligned}
&\rho\hat{\beta}_k\mu_C - \rho^2\hat{\beta}_k\left(a_C D_{id} + \int D_{ij}\hat{\beta}_j dj + z_i\right)\sigma_C^2 + \rho\mu_M - \rho^2\left(\int D_{ij}dj\right)\sigma_M^2 - \rho P_k = 0 \\
&\qquad\Leftrightarrow \hat{\beta}_k\mu_C + \hat{\beta}_k\frac{P_d - a_0 - a_C\mu_C}{a_C} + \mu_M - \rho\left(\int D_{ij}dj\right)\sigma_M^2 - P_k = 0.
\end{aligned} \tag{13}$$

Now, notice that the left-hand side of this equation is independent of i. Thus, we have that $\int D_{ij}dj$ is identical across the green and brown traders. Applying market clearing, we know that:

$$\sum_{i\in\{B,G\}} \lambda_i\left[\int D_{ij}dj\right] = \kappa.$$

These facts imply that, $\forall i \in [0,1]$, $\int D_{ij}dj = \kappa$. Note that, because equations (12) and (13) depend only on $\int D_{ij}dj$ and $\int D_{ij}\hat{\beta}_j dj$, any demand functions D_{id} and $\{D_{ij}\}_{j\in[0,1]}$ that satisfy these equations and clear the market will be consistent with an equilibrium, and generate identical prices and risk exposures for the investors. So, suppose that, $\forall i, j \in [0,1]$, $D_{ij} = \kappa$, which satisfies the market-clearing condition $\int D_{ij}dj = \kappa$. Moreover, note that this demand function ensures the variance of $D_{ij}\tilde{\beta}_j$ is uniformly bounded from above as a function of j, consistent with the conjectured equilibrium. Substituting into (12) and applying $\int \hat{\beta}_j dj = \mu_\beta$, we obtain:

$$-\rho\left(a_C D_{id} + \kappa\mu_\beta + z_i\right)\sigma_C^2 = \frac{P_d - a_0 - a_C\mu_C}{a_C}$$
$$\Leftrightarrow D_{id} = \frac{1}{a_C}\left[\frac{P_d - a_0 - a_C\mu_C}{-\rho a_C\sigma_C^2} - \kappa\mu_\beta - z_i\right].$$

Applying market clearing in the derivative, $\sum_i \lambda_i D_{id} = 0$, we obtain:

$$P_d = a_0 + a_C\mu_C - \rho a_C\sigma_C^2\left(\kappa\mu_\beta + \bar{z}\right),$$

and:

$$D_{id} = \frac{1}{a_C}\left[\frac{a_0 + a_C\mu_C - \rho a_C\sigma_C^2\left(\kappa\mu_\beta + \bar{z}\right) - a_0 - a_C\mu_C}{-\rho a_C\sigma_C^2} - \kappa\mu_\beta - z_i\right] = \frac{\bar{z} - z_i}{a_C}.$$

Now, let ω denote a realization of the random variables $\left\{\{\tilde{\alpha}_j\}, \left\{\tilde{\beta}_j\right\}, \tilde{F}_C, \tilde{F}_M\right\}$ and let $Q_i(\omega)$ denote investor i's marginal utility, $-\exp\left(-\rho w_i\right)$, conditional on ω. Then, note that:

$$\begin{aligned}
Q_i(\omega) &= -\exp\left(-\rho D_{id}\left(x_d - P_d\right) - \rho\int D_{ij}\left(\alpha_j + \beta_j F_C + F_M - P_j\right)dj - \rho z_i F_C - \rho\kappa\int P_j dj\right)\\
&= -\exp\left(-\rho\frac{\bar{z} - z_i}{a_C}\left(a_0 + a_C F_C - P_d\right) - \rho\int \kappa\left(\alpha_j + \beta_j F_C + F_M\right)dj - \rho z_i F_C\right)\\
&= -\exp\left(-\rho\frac{\bar{z} - z_i}{a_C}\left(a_0 - P_d\right) - \rho\bar{z}F_C - \rho\int \kappa\left(\alpha_j + \beta_j F_C + F_M\right)dj\right).
\end{aligned}$$

Hence, we have that:

$$\frac{Q_G(\omega)}{Q_B(\omega)} = \exp\left(\rho\left(a_0 - P_d\right)\left(\frac{z_G - z_B}{a_C}\right)\right),$$

which does not depend upon ω. This implies that investors' marginal utilities are equalized across states, which in turn implies that Pareto optimality is achieved. Finally, we can substitute into investors' ex-ante expected utility to obtain that:

$$\begin{aligned}
&EU_i\\
&= \exp\left(-\rho\left(\kappa\mu_\beta + \bar{z}\right)\mu_C + \frac{\rho^2}{2}\left(\bar{z} + \kappa\mu_\beta\right)^2\sigma_C^2 - \rho\kappa\left(\mu_M - \frac{\rho\kappa\sigma_M^2}{2}\right) + \rho\left(\bar{z} - z_i\right)\left(\mu_C - \rho\sigma_C^2\left(\kappa\mu_\beta + \bar{z}\right)\right)\right)\\
&= \exp\left(-\rho\left(\kappa\mu_\beta + z_i\right)\left(\mu_C - \frac{\rho\sigma_C^2}{2}\left(\kappa\mu_\beta + \bar{z}\right)\right) - \rho\kappa\left(\mu_M - \frac{\rho\kappa\sigma_M^2}{2}\right) - \frac{\rho^2\sigma_C^2}{2}\left(\kappa\mu_\beta + \bar{z}\right)\left(\bar{z} - z_i\right)\right)
\end{aligned}$$

Proof of Remark 2

This result follows in a very similar fashion to the case in which short-sale constraints do not bind in the proof of Proposition 1 below, and so I exclude the details.

Proof of Proposition 1, Proposition 2, Corollary 1, and Corollary 2

These results are closely related and not easily proved sequentially. Thus, I proceed in a series of steps. First, I show that Condition 1 implies an equilibrium in which short-sales constraints do not bind exists. Then, I derive prices in such an equilibrium and show it is efficient. Next, in the most significant part of the proof, I verify Proposition 2, Corollary 1, Corollary 2, and Corollary 4 by showing that, when Condition 1 is violated the equilibrium cannot be efficient, deriving the form of the equilibrium, and deriving prices in the equilibrium. Finally, I return to show that, when Condition 1 holds, there do not exist inefficient equilibria, which verifies that this condition is not only necessary, but also sufficient for the equilibrium to be efficient.

Step 1: Proof that Condition 1 implies an equilibrium in which short-sale constraints do not bind exists

Note that investor i's expected utility given a demand function $\{D_{ij}\}$ satisfies:

$$\begin{aligned} EU_i &\equiv -\frac{1}{\rho}\mathbb{E}_i\left[\exp\left(-\rho\int D_{ij}\left(\tilde{\alpha}_j+\tilde{\beta}_jF_C+F_M-P_j\right)dj-\rho z_iF_C-\rho\kappa\int P_jdj\right)\right] \\ &= -\frac{1}{\rho}\mathbb{E}_i\left[\exp\left(-\rho\int D_{ij}\left(\tilde{\beta}_jF_C+F_M-P_j\right)dj-\rho z_iF_C-\rho\kappa\int P_jdj\right)\right] \\ &\propto -\frac{1}{\rho}\mathbb{E}_i\left\{\mathbb{E}_i\left[\exp\left(-\rho\int D_{ij}\left(\tilde{\beta}_jF_C+F_M-P_j\right)dj-\rho z_iF_C\right)\mid\left\{\tilde{\beta}_j\right\}_{j\in[0,1]}\right]\right\} \\ &= -\frac{1}{\rho}\mathbb{E}_i\left[\exp\left(\begin{array}{c} -\rho\left(\int D_{ij}\tilde{\beta}_jdj+z_i\right)\mu_C+\frac{\rho^2}{2}\left(\int D_{ij}\tilde{\beta}_jdj+z_i\right)^2\sigma_C^2 \\ -\rho\left(\int D_{ij}dj\right)\mu_M+\frac{\rho^2}{2}\left(\int D_{ij}dj\right)^2\sigma_M^2+\rho\int D_{ij}P_jdj \end{array}\right)\right]. \end{aligned} \tag{14}$$

Now, note that, due to the presence of short-sale constraints, we have that $D_{ij}\geq 0$. Note further that $D_{ij}\leq\frac{\kappa}{\min(\lambda_B,\lambda_G)}$ since (i) I focus on equilibria where investors within the two groups behave symmetrically, and (ii) by market clearing, $\int D_{ij}dj=\kappa$, and so green (brown) investors' demands cannot exceed $\frac{\kappa}{\lambda_G}$ ($\frac{\kappa}{\lambda_B}$). Thus, the variance of $D_{ij}\tilde{\beta}_j$ is uniformly bounded from above, and we can follow the same steps as in the previous proof to arrive at:

$$\int D_{ij}\tilde{\beta}_jdj=\int\mathbb{E}\left[D_{ij}\tilde{\beta}_j\right]dj=\int D_{ij}\hat{\beta}_jdj.$$

Substituting this into (14), we obtain:

$$EU_i \propto -\frac{1}{\rho}\exp\left(\begin{array}{c} -\rho\left(\int D_{ij}\hat{\beta}_j dj + z_i\right)\mu_C + \frac{\rho^2}{2}\left(\int D_{ij}\hat{\beta}_j dj + z_i\right)^2\sigma_C^2 \\ -\rho\left(\int D_{ij}dj\right)\mu_M + \frac{\rho^2}{2}\left(\int D_{ij}dj\right)^2\sigma_M^2 + \rho\int D_{ij}P_j dj \end{array}\right).$$

Differentiating this expression, we obtain:

$$\frac{\partial EU_i}{\partial D_{ik}} = \left(\hat{\beta}_k\mu_C - \rho\hat{\beta}_k\left(\int D_{ij}\hat{\beta}_j dj + z_i\right)\sigma_C^2 + \mu_M - \rho\left(\int D_{ij}dj\right)\sigma_M^2 - P_k\right)\exp\left(...\right).$$

This has the sign of:

$$\Delta_i\left(k\right) \equiv \hat{\beta}_k\mu_C + \mu_M - P_k - \rho\hat{\beta}_k\left(\int D_{ij}\hat{\beta}_j dj + z_i\right)\sigma_C^2 - \rho\left(\int D_{ij}dj\right)\sigma_M^2.$$

Now, there exists an equilibrium in which investors' short-sale constraints do not bind if and only if there exist long-only portfolios $\left\{D_{Gj}, D_{Bj}\right\}_{j\in[0,1]}$ such that $\forall i, k$, $\Delta_i\left(k\right) = 0$, and the market clears in each stock. I next derive necessary and sufficient conditions on these portfolios for this to hold. Integrating the condition $\Delta_i\left(k\right) = 0$ over all firms gives:

$$\begin{aligned} 0 = \int\Delta_i\left(k\right)dk &= \int\left\{\hat{\beta}_k\mu_C + \mu_M - P_k - \rho\hat{\beta}_k\left(\int D_{ij}\hat{\beta}_j dj + z_i\right)\sigma_C^2 - \rho\left(\int D_{ij}dj\right)\sigma_M^2\right\}dk \\ \Leftrightarrow 0 &= \mu_\beta\mu_C + \mu_M - \int P_j dj - \rho\mu_\beta\left(\int D_{ij}\hat{\beta}_j dj + z_i\right)\sigma_C^2 - \rho\left(\int D_{ij}dj\right)\sigma_M^2. \end{aligned} \tag{15}$$

Let $\Lambda_i \equiv \int D_{ij}\hat{\beta}_j dj$ and $\Omega_i \equiv \int D_{ij}dj$. Averaging (15) over investors, we obtain:

$$\begin{aligned} 0 &= \sum_{i\in\{G,B\}}\lambda_i\int\Delta_i\left(j\right)dj \\ &= \sum_{i\in\{G,B\}}\lambda_i\left[\mu_\beta\mu_C + \mu_M - \int P_j dj - \rho\mu_\beta\left(\Lambda_i + z_i\right)\sigma_C^2 - \rho\Omega_i\sigma_M^2\right] \\ &= \mu_\beta\mu_C + \mu_M - \int P_j dj - \rho\mu_\beta\sigma_C^2\sum_{i\in\{G,B\}}\lambda_i\left(\Lambda_i + z_i\right) - \rho\sigma_M^2\sum_{i\in\{G,B\}}\lambda_i\Omega_i. \end{aligned}$$

We can apply the market-clearing condition at the market level $\sum_{i\in\{G,B\}}\lambda_i\Omega_i = \kappa$ to obtain:

$$\int P_j dj = \mu_\beta\mu_C + \mu_M - \rho\mu_\beta\sigma_C^2\sum_{i\in\{G,B\}}\lambda_i\left(\Lambda_i + z_i\right) - \rho\kappa\sigma_M^2. \tag{16}$$

Moreover, note that:

$$\begin{aligned}\sum_{i\in\{G,B\}}\lambda_i\left(\Lambda_i+z_i\right)&=\lambda_B\int D_{Bj}\hat{\beta}_j dj+\lambda_G\int D_{Gj}\hat{\beta}_j dj+\bar{z}\\&=\int\left(\lambda_B D_{Bj}+\lambda_G D_{Gj}\right)\hat{\beta}_j dj+\bar{z}\\&=\kappa\mu_\beta+\bar{z},\end{aligned}\tag{17}$$

where I applied the market-clearing condition at the stock level $\lambda_B D_{Bj}+\lambda_G D_{Gj}=\kappa$. Substituting (16) and (17) into (15), we obtain:

$$\begin{aligned}&0=\mu_\beta\mu_C+\mu_M-\left(\mu_\beta\mu_C+\mu_M-\rho\mu_\beta\sigma_C^2\left(\kappa\mu_\beta+\bar{z}\right)-\rho\kappa\sigma_M^2\right)-\rho\mu_\beta\sigma_C^2\left(\Lambda_i+z_i\right)-\rho\Omega_i\sigma_M^2\\\Leftrightarrow\ &0=\mu_\beta\sigma_C^2\left(\kappa\mu_\beta-\Lambda_i+\bar{z}-z_i\right)+\left(\kappa-\Omega_i\right)\sigma_M^2.\end{aligned}\tag{18}$$

Next, weighting the investors' first-order conditions in each stock by the stock's expected climate beta $\hat{\beta}_j$ and again integrating over firms, we obtain:

$$\begin{aligned}0&=\int\hat{\beta}_j\Delta_i\left(j\right)dj\\&=\int\hat{\beta}_j*\left(\hat{\beta}_j\mu_C+\mu_M-P_j-\rho\hat{\beta}_j\left(\Lambda_i+z_i\right)\sigma_C^2-\rho\Omega_i\sigma_M^2\right)dj\\&=\int\left(\hat{\beta}_j^2\mu_C+\hat{\beta}_j\left(\mu_M-P_j\right)-\rho\hat{\beta}_j^2\left(\Lambda_i+z_i\right)\sigma_C^2-\rho\hat{\beta}_j\Omega_i\sigma_M^2\right)dj\\&=\left(\mu_\beta^2+\sigma_\beta^2\right)\mu_C+\mu_\beta\mu_M-\int P_j\hat{\beta}_j dj-\rho\left(\mu_\beta^2+\sigma_\beta^2\right)\left(\Lambda_i+z_i\right)\sigma_C^2-\rho\mu_\beta\Omega_i\sigma_M^2.\end{aligned}\tag{19}$$

Averaging this condition over investors and applying equation (17) together with the market-clearing condition $\sum_{i\in\{G,B\}}\lambda_i\Omega_i=\kappa$ yields:

$$\begin{aligned}&0=\sum_{i\in\{G,B\}}\lambda_i\int\hat{\beta}_j\Delta_i\left(j\right)dj\\\Leftrightarrow\ &0=\sum_{i\in\{G,B\}}\lambda_i\left[\left(\mu_\beta^2+\sigma_\beta^2\right)\mu_C+\mu_\beta\mu_M-\int P_j\hat{\beta}_j dj-\rho\left(\mu_\beta^2+\sigma_\beta^2\right)\left(\Lambda_i+z_i\right)\sigma_C^2-\rho\mu_\beta\Omega_i\sigma_M^2\right]\\\Leftrightarrow\ &\int P_j\hat{\beta}_j dj=\left(\mu_\beta^2+\sigma_\beta^2\right)\mu_C+\mu_\beta\mu_M-\rho\left(\mu_\beta^2+\sigma_\beta^2\right)\left(\kappa\mu_\beta+\bar{z}\right)\sigma_C^2-\rho\kappa\mu_\beta\sigma_M^2.\end{aligned}\tag{20}$$

Substituting (20) into (19) yields:

$$\left(\mu_\beta^2+\sigma_\beta^2\right)\left(\kappa\mu_\beta-\Lambda_i+\bar{z}-z_i\right)\sigma_C^2+\mu_\beta\left(\kappa-\Omega_i\right)\sigma_M^2=0.\tag{21}$$

In sum, from equations (18) and (21), we have that $\{\Lambda_i, \Omega_i\}$ must satisfy the following system of equations:

$$\begin{pmatrix} \left(\mu_\beta^2 + \sigma_\beta^2\right)\sigma_C^2 & \mu_\beta \sigma_M^2 \\ \mu_\beta \sigma_C^2 & \sigma_M^2 \end{pmatrix} \begin{pmatrix} \kappa\mu_\beta - \Lambda_i + \bar{z} - z_i \\ \kappa - \Omega_i \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

Note that:

$$\det \begin{pmatrix} \left(\mu_\beta^2 + \sigma_\beta^2\right)\sigma_C^2 & \mu_\beta \sigma_M^2 \\ \mu_\beta \sigma_C^2 & \sigma_M^2 \end{pmatrix} = \sigma_\beta^2 \sigma_M^2 \sigma_C^2 \neq 0,$$

and so the unique solution $\{\Lambda_i, \Omega_i\}$ to this system satisfies:

$$\begin{aligned} \Lambda_i &= \kappa\mu_\beta + \bar{z} - z_i; \\ \Omega_i &= \kappa. \end{aligned} \tag{22}$$

I next argue that if we can find a portfolio $\{D_{Gj}\}_{j\in\{0,1\}}$ with $D_{Gj} \in \left[0, \frac{\kappa}{\lambda_G}\right]$ that solves this system for green investors, then the portfolio that each brown investor must hold to clear the market also satisfies this system for brown investors and does not involve shorting. Thus, for an equilibrium in which short-sales constraints do not bind to exist, it is necessary and sufficient that we can find such a portfolio. Note that, when each green investor holds the portfolio $\{D_{Gj}\}_{j\in\{0,1\}}$, market clearing in stock j yields:

$$\begin{aligned} &\lambda_G D_{Gj} + \lambda_B D_{Bj} = \kappa \\ \Leftrightarrow\ & D_{Bj} = \frac{\kappa - \lambda_G D_{Gj}}{\lambda_B}. \end{aligned}$$

Given the assumption that $D_{Gj} \leq \frac{\kappa}{\lambda_G}$, this is non-negative. Now, note that:

$$\begin{aligned} \Omega_B &= \int \frac{\kappa - \lambda_G D_{Gj}}{\lambda_B} dj \\ &= \frac{\kappa - \lambda_G \Omega_G}{1 - \lambda_G} = \kappa. \end{aligned}$$

Moreover,

$$\begin{aligned}\Lambda_B &= \int \frac{\kappa - \lambda_G D_{Gj}}{\lambda_B} \hat{\beta}_j dj \\ &= \frac{\kappa\mu_\beta - \lambda_G \Lambda_G}{\lambda_B} \\ &= \frac{\kappa\mu_\beta - \lambda_G \left(\kappa\mu_\beta + \lambda_G z_G + \lambda_B z_B - z_G\right)}{\lambda_B} \\ &= \kappa\mu_\beta + \bar{z} - z_B,\end{aligned}$$

as desired. The next lemma verifies that the condition in the statement of the proposition ensures that we can find such a portfolio $\{D_{Gj}\}_{j\in\{0,1\}}$.

Lemma 1. *Suppose that* $\frac{1}{\lambda_B}\int_{\lambda_G}^{1} \hat{\beta}_j dj - \frac{1}{\lambda_G}\int_0^{\lambda_G} \hat{\beta}_j dj \geq \frac{z_G - z_B}{\kappa}$*, Then, there exists a portfolio* $\left\{D^*_{Gj}\right\}_{j\in[0,1]}$ *such that:*

$$\begin{aligned} &\textit{(i)}\ D^*_{Gj} \in \left[0, \frac{\kappa}{\lambda_G}\right]; \\ &\textit{(ii)}\ \int D^*_{Gj} dj = \kappa; \\ &\textit{(iii)}\ \int D^*_{Gj} \hat{\beta}_j dj = \kappa\mu_\beta + \bar{z} - z_G. \end{aligned}$$

Proof. Note that the portfolio $\left\{D^L_{Gj}\right\}$ that minimizes $\int D_{Gj}\hat{\beta}_j dj$ subject to the conditions $\int D_{Gj} dj = \kappa;\ D_{Gj} \in \left[0, \frac{\kappa}{\lambda_G}\right]$ satisfies:

$$D^L_{Gj} = \begin{cases} \frac{\kappa}{\lambda_G} & \text{for } j < \lambda_G \\ 0 & \text{for } j > \lambda_G \end{cases}.$$

This is easily verified by setting up the Lagrangian and showing that the first-order condition is monotonic in $\hat{\beta}_j$. Let:

$$\Lambda^L \equiv \int D^L_{Gj}\hat{\beta}_j dj = \frac{\kappa}{\lambda_G}\int_0^{\lambda_G} \hat{\beta}_j dj.$$

I next show that, for any $\Lambda \in \left[\Lambda^L, \kappa\mu_\beta\right]$, there exists a portfolio $D^*_{Gj}(\Lambda)$ that sets $\int D^*_{Gj}(\Lambda)\,\hat{\beta}_j dj = \Lambda$ and satisfies conditions (i) and (ii). Let:

$$D^*_{Gj}(\Lambda) \equiv \frac{\Lambda - \Lambda^L}{\kappa\mu_\beta - \Lambda^L}\kappa + \left(1 - \frac{\Lambda - \Lambda^L}{\kappa\mu_\beta - \Lambda^L}\right) D^L_{Gj}.$$

Clearly, this portfolio satisfies condition (i). To see that it satisfies condition (ii), note that:

$$\begin{aligned}\int D_{Gj}^{*}(\Lambda)\,dj &= \frac{\Lambda-\Lambda^{L}}{\kappa\mu_{\beta}-\Lambda^{L}}\kappa+\left(1-\frac{\Lambda-\Lambda^{L}}{\kappa\mu_{\beta}-\Lambda^{L}}\right)\int D_{Gj}^{L}dj\\ &= \frac{\Lambda-\Lambda^{L}}{\kappa\mu_{\beta}-\Lambda^{L}}\kappa+\left(1-\frac{\Lambda-\Lambda^{L}}{\kappa\mu_{\beta}-\Lambda^{L}}\right)\kappa=\kappa.\end{aligned}$$

Finally,

$$\int D_{Gj}^{*}(\Lambda)\,\hat{\beta}_{j}dj=\frac{\Lambda-\Lambda^{L}}{\kappa\mu_{\beta}-\Lambda^{L}}\kappa\mu_{\beta}+\left(1-\frac{\Lambda-\Lambda^{L}}{\kappa\mu_{\beta}-\Lambda^{L}}\right)\Lambda^{L}=\Lambda.$$

These results imply that, to prove the lemma, it is sufficient to show that $\kappa\mu_{\beta}+\bar{z}-z_{G}\in\left[\Lambda^{L},\kappa\mu_{\beta}\right]$. Since $z_{G}>\bar{z}$, we immediately have that $\kappa\mu_{\beta}+\bar{z}-z_{G}<\kappa\mu_{\beta}$. Now, note:

$$\begin{aligned}& 0\leq\kappa\mu_{\beta}+\bar{z}-z_{G}-\Lambda^{L}\\ \Leftrightarrow\ & 0\leq\kappa\mu_{\beta}+\bar{z}-z_{G}-\frac{\kappa}{\lambda_{G}}\int_{0}^{\lambda_{G}}\hat{\beta}_{j}dj\\ \Leftrightarrow\ & 0\leq\kappa\left(\int_{0}^{\lambda_{G}}\hat{\beta}_{j}dj+\int_{\lambda_{G}}^{1}\hat{\beta}_{j}dj\right)-\frac{\kappa}{\lambda_{G}}\int_{0}^{\lambda_{G}}\hat{\beta}_{j}dj+\lambda_{B}\left(z_{B}-z_{G}\right)\\ \Leftrightarrow\ & \frac{z_{G}-z_{B}}{\kappa}\leq\left(\frac{\lambda_{G}-1}{\lambda_{B}\lambda_{G}}\right)\int_{0}^{\lambda_{G}}\hat{\beta}_{j}dj+\frac{1}{\lambda_{B}}\int_{\lambda_{G}}^{1}\hat{\beta}_{j}dj\\ \Leftrightarrow\ & \frac{z_{G}-z_{B}}{\kappa}\leq\frac{1}{\lambda_{B}}\int_{\lambda_{G}}^{1}\hat{\beta}_{j}dj-\frac{1}{\lambda_{G}}\int_{0}^{\lambda_{G}}\hat{\beta}_{j}dj,\end{aligned}$$

which is the condition in the statement of the lemma. □

Step 2: Derivation of equilibrium prices and efficiency in an equilibrium where short-sales constraints do not bind

We can return to the investors' first-order condition to obtain the equilibrium prices:

$$\begin{aligned}& 0=\Delta_{i}(k)=\hat{\beta}_{k}\mu_{C}+\mu_{M}-P_{k}-\rho\hat{\beta}_{k}\left(\int D_{ij}\hat{\beta}_{j}dj+z_{i}\right)\sigma_{C}^{2}-\rho\left(\int D_{ij}dj\right)\sigma_{M}^{2}\\ \Leftrightarrow\ & P_{k}=\hat{\beta}_{k}\mu_{C}+\mu_{M}-\rho\hat{\beta}_{k}\left(\int D_{ij}\hat{\beta}_{j}dj+z_{i}\right)\sigma_{C}^{2}-\rho\left(\int D_{ij}dj\right)\sigma_{M}^{2}\\ \Leftrightarrow\ & P_{k}=\hat{\beta}_{k}\mu_{C}+\mu_{M}-\rho\hat{\beta}_{k}\left(\kappa\mu_{\beta}+\bar{z}\right)\sigma_{C}^{2}-\rho\kappa\sigma_{M}^{2}.\end{aligned}$$

To see that this equilibrium achieves Pareto optimality, let ω denote a realization of the random variables $\{\{\tilde{\alpha}_{j}\},\{\beta_{j}\},F_{C},F_{M}\}$ and let $Q_{i}(\omega)$ denote investor i's marginal utility,

$-\exp(-\rho w_i)$, conditional on ω. Then, note that:

$$\begin{aligned}
Q_i(\omega) &= -\exp\left(-\rho\int D^*_{ij}\left(\alpha_j+\beta_j F_C+F_M-P_j\right)dj-\rho z_i F_C-\rho\kappa\int P_j dj\right)\\
&= \exp\left(-\rho F_C\left(\int D^*_{ij}\beta_j dj+z_i\right)-\rho\int D^*_{ij}\left(F_M-P_j\right)dj-\rho\kappa\int P_j dj\right)\\
&= \exp\left(-\rho F_C\left(\int D^*_{ij}\hat{\beta}_j dj+z_i\right)-\rho\kappa F_M+\rho\int\left(D^*_{ij}-\kappa\right)P_j dj\right)\\
&= \exp\left(-\rho F_C\left(\kappa\mu_\beta+\bar{z}\right)-\rho\kappa F_M+\rho\int\left(D^*_{ij}-\kappa\right)P_j dj\right).
\end{aligned}$$

Hence, we have that:

$$\frac{Q_G(\omega)}{Q_B(\omega)}=\exp\left(\rho\int\left(D^*_{Gj}-D^*_{Bj}\right)P_j dj\right),$$

which does not depend upon ω. This implies that investors' marginal utilities are equalized across states, which in turn implies that Pareto optimality is achieved. Finally, turning to expected utilities, we have:

$$\begin{aligned}
EU_i &= -\frac{1}{\rho}\exp\begin{pmatrix} -\rho\left(\int D_{ij}\hat{\beta}_j dj+z_i\right)\mu_C+\frac{\rho^2}{2}\left(\int D_{ij}\hat{\beta}_j dj+z_i\right)^2\sigma_C^2 \\ -\rho\left(\int D_{ij}dj\right)\mu_M+\frac{\rho^2}{2}\left(\int D_{ij}dj\right)^2\sigma_M^2+\rho\int\left(D_{ij}-\kappa\right)P_j dj \end{pmatrix}\\
&= -\frac{1}{\rho}\exp\begin{pmatrix} -\rho\left(\kappa\mu_\beta+\bar{z}\right)\left(\mu_C-\frac{\rho\sigma_C^2}{2}\left(\kappa\mu_\beta+\bar{z}\right)\right)-\rho\kappa\left(\mu_M-\frac{\rho\kappa\sigma_M^2}{2}\right) \\ +\rho\left(\bar{z}-z_i\right)\left(\mu_C-\rho\left(\kappa\mu_\beta+\bar{z}\right)\sigma_C^2\right) \end{pmatrix},
\end{aligned}$$

which one can verify is identical to the expression obtained in Remark 1.

Step 3: Proof of Proposition 2, Corollary 1, Corollary 2, and Corollary 4

I start by establishing three lemmas, which establish the core features of the equilibrium when Condition 1 does not hold.

Lemma 2. *Suppose that Condition 1 is violated. Then, the set*

$$S\equiv\{k:D_{Gk}>0 \text{ and } D_{Bk}>0\}$$

has measure zero, i.e., only one of the investor groups holds any given stock except potentially on a set of measure zero.

Proof. Suppose by contradiction that S contains an open interval I. Then, by the Kuhn-

Tucker theorem, $\forall k \in I$, $\Delta_B(k) = \Delta_G(k) = 0$. Simplifying, we obtain, $\forall k \in I$:

$$\begin{aligned} \Delta_B(k) &= \Delta_G(k) \\ \Leftrightarrow \hat{\beta}_k(\Lambda_B + z_B)\,\sigma_C^2 + \Omega_B\sigma_M^2 &= \hat{\beta}_k(\Lambda_G + z_G)\,\sigma_C^2 + \Omega_G\sigma_M^2. \end{aligned} \tag{23}$$

Differentiating with respect to k and dividing by σ_C^2, we arrive at:

$$\Lambda_B + z_B = \Lambda_G + z_G. \tag{24}$$

Substituting (24) into expression (23), we obtain:

$$\begin{aligned} \hat{\beta}_k(\Lambda_G + z_G)\,\sigma_C^2 + \Omega_B\sigma_M^2 &= \hat{\beta}_k(\Lambda_G + z_G)\,\sigma_C^2 + \Omega_G\sigma_M^2 \\ \Leftrightarrow \Omega_B &= \Omega_G. \end{aligned} \tag{25}$$

Equation (25) together with the market-clearing condition implies $\Omega_B = \Omega_G = \kappa$. Now, recall from the proof of Lemma 1 that the portfolio $\left\{D_{Gj}^L\right\}$ with $D_{Gj}^L \in \left[0, \frac{\kappa}{\lambda_G}\right]$ that minimizes $\int D_{Gj}\hat{\beta}_j dj$ subject to $\int D_{Gj}dj = \kappa$ satisfies:

$$D_{Gj}^L = \begin{cases} \frac{\kappa}{\lambda_G} & \text{for } j < \lambda_G \\ 0 & \text{for } j > \lambda_G \end{cases}.$$

A similar argument yields that the portfolio $\left\{D_{Bj}^H\right\}$ with $D_{Bj}^H \in \left[0, \frac{\kappa}{\lambda_B}\right]$ that maximizes $\int D_{Bj}\hat{\beta}_j dj$ subject to $\int D_{Bj}dj = \kappa$ satisfies:

$$D_{Bj}^H = \begin{cases} 0 & \text{for } j < \lambda_G \\ \frac{\kappa}{\lambda_B} & \text{for } j > \lambda_G \end{cases}.$$

Combining this with (24), we obtain:

$$\begin{aligned} 0 = \Lambda_B - \Lambda_G + z_B - z_G &< \int D_{Bj}^H\hat{\beta}_j dj - \int D_{Gj}^L\hat{\beta}_j dj + z_B - z_G \\ &= \frac{\kappa}{\lambda_G}\int_0^{\lambda_G}\hat{\beta}_j dj - \frac{\kappa}{\lambda_B}\int_{\lambda_G}^1\hat{\beta}_j dj + z_B - z_G. \end{aligned}$$

This contradicts the assumption that $\frac{1}{\lambda_B}\int_{\lambda_G}^1\hat{\beta}_j dj - \frac{1}{\lambda_G}\int_0^{\lambda_G}\hat{\beta}_j dj < \frac{z_G - z_B}{\kappa}$. □

Lemma 3. *Suppose that Condition 1 is violated. In equilibrium, green investors must remain weakly more exposed to climate risk than brown investors, i.e.,* $\Lambda_G + z_G \geq \Lambda_B + z_B$.

Proof. We have that:

$$\begin{aligned}
\frac{\partial}{\partial k}\left[\Delta_G(k)-\Delta_B(k)\right] &= \frac{\partial}{\partial k}\left[\begin{array}{c}\hat{\beta}_k\mu_C+\mu_M-P_k-\rho\hat{\beta}_k\left(\Lambda_G+z_G\right)\sigma_C^2-\rho\Omega_G\sigma_M^2\\ -\left(\hat{\beta}_k\mu_C+\mu_M-P_k-\rho\hat{\beta}_k\left(\Lambda_B+z_B\right)\sigma_C^2-\rho\Omega_B\sigma_M^2\right)\end{array}\right]\\
&= \frac{\partial}{\partial k}\left[-\hat{\beta}_k\left(\Lambda_G+z_G-\left(\Lambda_B+z_B\right)\right)\sigma_C^2-\left(\Omega_G-\Omega_B\right)\sigma_M^2\right]\\
&\propto \frac{\partial\hat{\beta}_k}{\partial k}\left[\left(\Lambda_B+z_B\right)-\left(\Lambda_G+z_G\right)\right]. \qquad (26)
\end{aligned}$$

So, suppose by contradiction that $\Lambda_G+z_G<\Lambda_B+z_B$. Then, from (26), we have that $\frac{\partial[\Delta_G(k)-\Delta_B(k)]}{\partial k}>0$. Note the previous lemma, together with the Kuhn-Tucker theorem, implies that, $\forall j\in[0,1]$, either (i) $\Delta_B(j)\leq 0$ and $\Delta_G(j)=0$, or (ii) $\Delta_B(j)=0$ and $\Delta_G(j)\leq 0$. Therefore, $\frac{\partial[\Delta_G(k)-\Delta_B(k)]}{\partial k}>0$ implies that the green investors hold all stocks *above* a cutoff $\tau\in[0,1]$, i.e.,

$$\Lambda_G=\frac{\kappa}{\lambda_G}\int_\tau^1\hat{\beta}_j dj \quad \text{and} \quad \Lambda_B=\frac{\kappa}{\lambda_B}\int_0^\tau\hat{\beta}_j dj. \qquad (27)$$

As an intermediate step towards deriving a contradiction, I next show that this implies $\hat{\beta}_\tau>0$. Applying the original assumption that $\Lambda_G+z_G<\Lambda_B+z_B$ and (27), we obtain:

$$\begin{aligned}
\Lambda_G+z_G<\Lambda_B+z_B &\Leftrightarrow \Lambda_G-\Lambda_B<z_B-z_G\\
&\Rightarrow \Lambda_G-\Lambda_B<0\\
&\Leftrightarrow \kappa\left[\frac{1}{\lambda_G}\int_0^\tau\hat{\beta}_j dj-\frac{1}{\lambda_B}\int_\tau^1\hat{\beta}_j dj\right]<0\\
&\Leftrightarrow \frac{1}{\lambda_G}\int_0^\tau\hat{\beta}_j dj-\frac{1}{\lambda_B}\int_\tau^1\hat{\beta}_j dj<0. \qquad (28)
\end{aligned}$$

Now, suppose by contradiction that $\hat{\beta}_\tau\leq 0$. Then, we would have that, $\forall t<\tau$, $\hat{\beta}_t<0$, and so:

$$\int_\tau^1\hat{\beta}_j dj>\int_\tau^1\hat{\beta}_j dj+\int_0^\tau\hat{\beta}_j dj=\mu_\beta.$$

Therefore, we have that $\frac{1}{\lambda_G}\int_\tau^1\hat{\beta}_j dj-\frac{1}{\lambda_B}\int_0^\tau\hat{\beta}_j dj>0$, which contradicts (28). Next, I argue that $\Omega_G<\Omega_B$, which is equivalent to:

$$\begin{aligned}
\Omega_G<\Omega_B &\Leftrightarrow \frac{\kappa\left(1-\tau\right)}{\lambda_G}<\frac{\kappa\tau}{\lambda_B}\\
&\Leftrightarrow \tau>\lambda_B.
\end{aligned}$$

To see this, note that, given that $\hat{\beta}_\tau > 0$, (28) implies:

$$\begin{aligned}
\frac{1}{\lambda_G}\int_\tau^1 \hat{\beta}_j dj - \frac{1}{\lambda_B}\int_0^\tau \hat{\beta}_j dj < 0 &\Rightarrow \frac{1}{\lambda_G}\int_\tau^1 \hat{\beta}_\tau dj - \frac{1}{\lambda_B}\int_0^\tau \hat{\beta}_\tau dj < 0 \\
&\Rightarrow \hat{\beta}_\tau \left(\frac{1-\tau}{\lambda_G} - \frac{\tau}{\lambda_B}\right) < 0 \\
&\Rightarrow \tau > \lambda_B,
\end{aligned}$$

Combining the facts that $\Lambda_G - \Lambda_B < z_B - z_G$ and $\Omega_G < \Omega_B$, we obtain, $\forall k \in [0,1]$:

$$\Delta_G(k) - \Delta_B(k) \propto -\hat{\beta}_k(\Lambda_G + z_G - (\Lambda_B + z_B))\,\sigma_C^2 - (\Omega_G - \Omega_B)\,\sigma_M^2,$$

which is strictly positive for all $\hat{\beta}_k \geq 0$. This implies that the green investors hold all stocks with $\hat{\beta}_k > 0$, i.e., $\hat{\beta}_\tau \leq 0$. But, this contradicts the previous finding that $\hat{\beta}_\tau > 0$. This concludes the proof. □

Lemma 4. *Suppose that Condition 1 is violated. Then, there exists a $T \in [0,1)$ such that brown investors hold all of stocks $j > T$ and green investors holds all of stocks $j < T$.*

Proof. First, I show that, if the brown investors hold stock l, then they also hold stock $h > l$. From (26), we have that $\frac{\partial[\Delta_G(k) - \Delta_B(k)]}{\partial k} < 0$. Therefore, if $\Delta_B(l) = 0$ and $\Delta_G(l) \leq 0$, then it must also be the case that $\Delta_B(h) = 0$ and $\Delta_G(h) < 0$. Together with the Kuhn-Tucker theorem, this verifies that if the brown investors hold stock l, then they also hold stock $h > l$. Finally, to see that we cannot have $T = 1$, note that, in this case, we would obtain that:

$$\Delta_G(k) - \Delta_B(k) \propto -\left(\lim_{t\to 1}\hat{\beta}_t\right)(\kappa\mu_\beta + z_G - z_B)\,\sigma_C^2 - \frac{\kappa}{\lambda_G}\sigma_M^2 < 0,$$

which implies brown investors must hold stocks j in a neighborhood of $j = 1$. □

I next determine the firms' prices in such an equilibrium. Note that, for $k > T$, we can rewrite $\Delta_B(k) = 0$ to obtain:

$$\begin{aligned}
P_k &= \hat{\beta}_k\mu_C + \mu_M - \rho\hat{\beta}_k\left(\int D_{Bj}\hat{\beta}_j dj + z_B\right)\sigma_C^2 - \rho\left(\int D_{Bj} dj\right)\sigma_M^2 \\
&= \hat{\beta}_k\mu_C + \mu_M - \rho\hat{\beta}_k\left(\frac{\kappa}{\lambda_B}\int_T^1 \hat{\beta}_j dj + z_B\right)\sigma_C^2 - \rho\frac{\kappa}{\lambda_B}(1-T)\,\sigma_M^2.
\end{aligned}$$

Similarly, for $k < T$, we can rewrite $\Delta_G(k) = 0$ to obtain:

$$\begin{aligned} P_k &= \hat{\beta}_k \mu_C + \mu_M - \rho \hat{\beta}_k \left(\int D_{Gj} \hat{\beta}_j dj + z_G \right) \sigma_C^2 - \rho \left(\int D_{Gj} dj \right) \sigma_M^2 \\ &= \hat{\beta}_k \mu_C + \mu_M - \rho \hat{\beta}_k \left(\frac{\kappa}{\lambda_G} \int_0^T \hat{\beta}_j dj + z_G \right) \sigma_C^2 - \rho \frac{\kappa}{\lambda_G} T \sigma_M^2. \end{aligned}$$

This verifies Corollary 4.

I next show that there is a unique $T \in [0,1)$ that corresponds to an equilibrium, and in the process, prove Corollaries 1 and 2. Let $H(t) : [0,1] \to \mathcal{R}$ denote the difference between the climate risk exposure of a green and a brown investor, given that the brown investors hold all stocks above cutoff t:

$$H(t) = \frac{\kappa}{\lambda_G} \int_0^t \hat{\beta}_j dj + z_G - \left(\frac{\kappa}{\lambda_B} \int_t^1 \hat{\beta}_j dj + z_B \right).$$

In equilibrium, we must have that $\Delta_B(k) \leq 0$ for $k \leq T$, which reduces as follows:

$$\begin{aligned} \Delta_B(k) &= \hat{\beta}_k \mu_C + \mu_M - \rho \hat{\beta}_k \left(\frac{\kappa}{\lambda_B} \int_T^1 \hat{\beta}_j dj + z_B \right) \sigma_C^2 - \rho \frac{\kappa}{\lambda_B} (1 - T) \sigma_M^2 - P_k \leq 0 \\ &\Leftrightarrow \hat{\beta}_k H(T) \sigma_C^2 + \kappa \left(\frac{T - \lambda_G}{\lambda_B \lambda_G} \right) \sigma_M^2 \leq 0. \end{aligned} \tag{29}$$

Similarly, we must have that $\Delta_G(k) \leq 0$ for $k > T$, which reduces as follows:

$$\begin{aligned} \Delta_G(k) &= \hat{\beta}_k \mu_C + \mu_M - \rho \hat{\beta}_k \left(\frac{\kappa}{\lambda_G} \int_0^T \hat{\beta}_j dj + z_G \right) \sigma_C^2 - \rho \frac{\kappa}{\lambda_G} T \sigma_M^2 - P_k \leq 0 \\ &\Leftrightarrow -\hat{\beta}_k H(T) \sigma_C^2 - \kappa \left(\frac{T - \lambda_G}{\lambda_B \lambda_G} \right) \sigma_M^2 \leq 0. \end{aligned} \tag{30}$$

From Lemmas 3 and 4, in searching for an equilibrium T, we can constrain our attention to $T \in H_+$, where:

$$H_+ \equiv \{ t : t \in [0,1), H(t) \geq 0 \}.$$

Observe that, if $T \in H_+$, then $\Delta_B(k)$ and $\Delta_G(k)$ are increasing and decreasing in k, respectively. Hence, if $\Delta_B(T) = \Delta_G(T) = 0$, then we will have that, $\forall k < T$, $\Delta_B(k) \leq 0$, and $\forall k > T$, $\Delta_G(k) \leq 0$. So, define the function $G(t) : (0,1) \to \mathcal{R}$:

$$G(t) \equiv \hat{\beta}_t H(t) \sigma_C^2 + \kappa \left(\frac{t - \lambda_G}{\lambda_B \lambda_G} \right) \sigma_M^2,$$

and notice that $G(T) = 0 \Leftrightarrow \Delta_B(T) = \Delta_G(T) = 0$. Thus, if we can find a $T \in H_+ \setminus 0$ such that $G(T) = 0$, then T will constitute an equilibrium in which both investor groups hold a strictly positive measure of stocks. It is helpful to establish the following lemma.

Lemma 5. *The set H_+ takes the form:*

$$H_+ = [0, h_L] \cup [h_U, 1), \tag{31}$$

where $h_L \geq 0$ and $h_U < 1$. Furthermore, $\hat{\beta}_t > 0$ for $t \in (h_U, 1]$ and, when $h_L > 0$, $\hat{\beta}_t < 0$ for $t \in [0, h_L)$.

Proof. Observe that:

$$H'(t) = \kappa \left(\frac{1}{\lambda_G} + \frac{1}{\lambda_B} \right) \hat{\beta}_t,$$

and so $\operatorname{sgn}(H'(t)) = \operatorname{sgn}\left(\hat{\beta}_t\right)$. Let t_0 denote the solution to $\hat{\beta}_{t_0} = 0$ if it exists and 0 otherwise, so that, for $t \in [0, 1)$,

$$\operatorname{sgn}(H'(t)) = \operatorname{sgn}(t - t_0). \tag{32}$$

The fact that $H'(\cdot)$ changes sign at most once implies that $H(\cdot)$ has at most two zeros on $[0, 1)$. Let h_L equal the lower zero of $H(\cdot)$ if $H(\cdot)$ has two zeros and 0 if $H(\cdot)$ has fewer than two zeroes. Likewise, let h_U equal the upper zero of $H(\cdot)$ if $H(\cdot)$ has at least one zero and let $h_U = 0$ if it has no zeroes. Because H' shifts from negative to positive, and because $H(1) = \frac{\kappa \mu_\beta}{\lambda_G} + z_G - z_B > 0$, we must have that, when $h_U > 0$, either (i) $H(t)$ crosses zero from below at h_U, or that (ii) $H(t)$ reaches its minimum at h_U, i.e., from (32), $h_U = t_0$. Likewise, $H(t)$ must cross zero from above at h_L when $h_L > 0$. This immediately implies that $H(t) \geq 0$ for $t \in [0, h_L] \cup [h_U, 1)$ and $H(t) < 0$ for $t \in (h_L, h_U)$, which verifies that H_+ takes the form in (31). Note further that because, when $h_L > 0$, $H'(h_L) < 0$, we must have $h_L < t_0$, and so $\hat{\beta}_t < 0$ for $t \in [0, h_L)$. Likewise, because $H'(h_U) \geq 0$, we must have $h_U \geq t_0$, with $h_U = t_0$ if and only if $H'(h_U) = 0$. Hence, $\hat{\beta}_t > 0$ for $t \in (h_U, 1)$. □

To complete the proof, I separately analyze the cases in which $\hat{\beta}_{\lambda_G} \geq 0$ and $\hat{\beta}_{\lambda_G} < 0$.

Case 1: $\boldsymbol{\hat{\beta}_{\lambda_G} \geq 0}$. I start by showing the following.

Lemma 6. *Suppose that $\hat{\beta}_{\lambda_G} \geq 0$. Then, $h_U \leq \lambda_G$, i.e., $[\lambda_G, 1) \subset H_+$.*

Proof. Note that:

$$H(\lambda_G) = \frac{\kappa}{\lambda_G} \int_0^{\lambda_G} \hat{\beta}_j dj - \frac{\kappa}{\lambda_B} \int_{\lambda_G}^1 \hat{\beta}_j dj + z_G - z_B.$$

This is positive because Condition 1 is violated. So, $\lambda_G \in H_+$, and to complete the proof, we need only show that $\lambda_G \notin [0, h_L]$. This is trivial when $h_L = 0$, and when $h_L > 0$, it follows by the assumption that $\hat{\beta}_{\lambda_G} \geq 0$, since this yields $\lambda_G \geq t_0 > h_L$. $\square$

Applying this lemma, for any $t > \lambda_G$, we have that:

$$G(t) \equiv \underbrace{\hat{\beta}_t}_{>0} \underbrace{H(t)}_{>0} \sigma_C^2 + \kappa \underbrace{\left(\frac{t - \lambda_G}{\lambda_B \lambda_G}\right)}_{\geq 0} \sigma_M^2 > 0, \tag{33}$$

and so, in searching for T such that $G(T) = 0$, we can constrain attention to searching for a $T \leq \lambda_G$. Next, observe that, if $t < h_L$, Lemma 5 implies:

$$G(t) \equiv \underbrace{\hat{\beta}_t}_{<0} \underbrace{H(t)}_{\geq 0} \sigma_C^2 + \kappa \underbrace{\left(\frac{t - \lambda_G}{\lambda_B \lambda_G}\right)}_{<0} \sigma_M^2 < 0,$$

so we can also constrain attention to searching for a $T \geq h_U$. Now, I break the analysis into two subcases.

Subcase 1: $\mathbf{h_U > 0}$, i.e., $\mathbf{H_+ \neq (0, 1)}$. In this case, note that:

$$G(\lambda_G) = \underbrace{\hat{\beta}_{\lambda_G}}_{\geq 0} \underbrace{H(\lambda_G)}_{>0} \sigma_C^2 \geq 0; \tag{34}$$

$$G(h_U) = \kappa \left(\frac{h_U - \lambda_G}{\lambda_B \lambda_G}\right) \sigma_M^2 < 0.$$

Thus, by the intermediate value theorem, there exists a $T \in (h_U, \lambda_G]$ with $G(T) = 0$, which constitutes an equilibrium. Moreover, notice that:

$$G'(t) = \left[\frac{\partial \hat{\beta}_t}{\partial t}\right] H(t) + \frac{\kappa\left(\hat{\beta}_t^2 + 1\right)}{\lambda_G \lambda_B}, \tag{35}$$

which is positive on H_+ and so is positive on $(h_U, \lambda_G]$. Thus, T is unique, which verifies that there is a unique equilibrium in which both investor groups hold a strictly positive measure of stocks. Note these arguments directly imply results (6) and (8) in Corollary 2 in this case. To see why, notice from (34) that $\hat{\beta}_{\lambda_G} = 0 \Rightarrow G(\lambda_G) = 0$, and so $T = \lambda_G$ is the unique equilibrium. Moreover, if $\hat{\beta}_{\lambda_G} > 0$, $G(\lambda_G) > 0$, and so the unique equilibrium T is strictly below λ_G.

Subcase 2: $\mathbf{h_U = 0}$, i.e., $\mathbf{H_+ = (0, 1)}$. In this case, note that:

$$\begin{aligned}\lim_{t\to 0} G(t) &= \left(\lim_{t\to 0} \hat{\beta}_t\right) H(0)\, \sigma_C^2 - \frac{\kappa\sigma_M^2}{\lambda_B} \\ &= \left(\lim_{t\to 0} \hat{\beta}_t\right) \left(z_G - z_B - \frac{\kappa}{\lambda_B}\mu_\beta\right) \sigma_C^2 - \frac{\kappa\sigma_M^2}{\lambda_B}. \end{aligned} \tag{36}$$

Suppose this is strictly negative. Then, note we again have that $G(\lambda_G) \geq 0$, and, given that $H_+ = (0, 1)$, $G'(t) > 0$ for all $t \in (0, 1)$. Thus, $G(\cdot)$ has a unique solution in $(0, \lambda_G]$, which corresponds to an equilibrium in which both investor groups hold a strictly positive measure of stocks. In contrast, if (36) is weakly positive, notice that:

$$\lim_{t\to 0} \Delta_B(t) = \left(\lim_{t\to 0} \hat{\beta}_t\right) H(0)\, \sigma_C^2 - \frac{\kappa\sigma_M^2}{\lambda_B} = \lim_{t\to 0} G(t) \geq 0.$$

Moreover, $\Delta_B(t)$ is increasing on $[0, 1]$, and so, for $t > 0$, $\Delta_B(t) > 0$. Likewise, since $\Delta_G(t) = -\Delta_B(t)$, we have that $\Delta_B(t) < 0$. Thus, in the unique equilibrium, brown investors hold the entire market. Note that Corollary 2 again holds in this case. When (36) is strictly negative, this corollary holds by the same exact reasoning as in the previous subcase. Next, it is easily seen that if (36) is weakly positive, then $\hat{\beta}_{\lambda_G} > 0$. Moreover, consistent with the corollary, we have that $T = 0 < \lambda_G$.

Case 2: $\boldsymbol{\hat{\beta}_{\lambda_G} < 0}$. Following the same initial steps as in the proof of Lemma 6, we have that $H(\lambda_G) > 0$. Therefore, applying Lemma 5, given that $\hat{\beta}_{\lambda_G} < 0$, we must have that $\lambda_G < h_L$. I first argue that we can constrain attention to looking for a solution $T \in H_+$ to $G(T) = 0$ to (λ_G, h_L). To see why, note that, for any $t > h_U > \lambda_G$, we have:

$$G(t) = \underbrace{\hat{\beta}_t}_{>0} \underbrace{H(t)}_{\geq 0} \sigma_C^2 + \kappa \underbrace{\left(\frac{t - \lambda_G}{\lambda_B \lambda_G}\right)}_{>0} \sigma_M^2 > 0.$$

Moreover, for any $t \leq \lambda_G$, because $\hat{\beta}_{\lambda_G} < 0$, we have:

$$G(t) = \underbrace{\hat{\beta}_t}_{<0} \underbrace{H(t)}_{>0} \sigma_C^2 + \kappa \underbrace{\left(\frac{t - \lambda_G}{\lambda_B \lambda_G}\right)}_{\leq 0} \sigma_M^2 < 0.$$

This proves the result in Corollary 2 that $\hat{\beta}_{\lambda_G} < 0 \Rightarrow T > \lambda_G$. Next, I verify that there is a unique $T \in (\lambda_G, h_L)$ that satisfies $G(T) = 0$. The previous equation demonstrates that $G(\lambda_G) < 0$; moreover, since $H(h_L) = 0$, we have:

$$G(h_L) = \kappa \left(\frac{h_L - \lambda_G}{\lambda_B \lambda_G} \right) \sigma_M^2 > 0.$$

Once again, by the intermediate value theorem, this implies that there exists a $T \in (\lambda_G, h_L)$ such that $G(T) = 0$. Furthermore, it is unique since $G'(t) > 0$ on H_+.

Finally, to see that Corollary 1 holds, notice that the only case that green investors do not participate is in Subcase 2 above, when:

$$\hat{\beta}_{\lambda_G} \geq 0; \tag{37}$$

$$\left(\lim_{t \to 0} \hat{\beta}_t \right) \left(z_G - z_B - \frac{\kappa}{\lambda_B} \mu_\beta \right) \sigma_C^2 - \frac{\kappa \sigma_M^2}{\lambda_B} > 0; \tag{38}$$

$$H_+ = [0, 1). \tag{39}$$

I now show these conditions are equivalent to the two conditions stated in the text, which are (38) and $H(0) = z_G - z_B - \frac{\kappa}{\lambda_B} \mu_\beta > 0$. Note the final two conditions above immediately imply the conditions in the text, and so we only need to show that the converse holds. Note that the two conditions in the text immediately imply that $\lim_{t \to 0} \hat{\beta}_t > 0$. This, in turn, implies (37). Moreover, recall that $H(t)$ is increasing when $\hat{\beta}_t > 0$, and so, given that $\lim_{t \to 0} \hat{\beta}_t > 0$, H is increasing on $(0, 1)$. Together with $H(0) > 0$, this implies (39).

Step 4: Proof that Condition 1 implies that inefficient equilibria do not exist

From the previous step, in an inefficient equilibrium, either (i) brown investors hold all stocks and $\lim_{t \to 0} G(t) \geq 0$, or green investors hold stocks in $[0, T)$ and brown investors hold stocks in $(T, 1]$, where $T \in H_+$ solves $G(T) = 0$. Note that, if, Condition 1 holds as an equality, then $G(\lambda_G) = 0$, and so there is a unique equilibrium of the latter form with $T = \lambda_G$. In this equilibrium, both groups' climate and market exposures are clearly equalized, so that the equilibrium is efficient and the proof is complete in this case. Thus, suppose now that Condition 1 holds as a strict inequality. To complete the proof, it suffices to show that, in this case, (i) $\lim_{t \to 0} G(t) < 0$, so there cannot exist an equilibrium in which brown investors hold all stocks, and that (ii) there is no solution $T \in H_+$ to $G(T) = 0$. We now have:

$$H(\lambda_G) < 0,$$

i.e., $\lambda_G \in (h_L, h_U)$. Hence, for $t \in (0, h_L]$, we have that:

$$G(t) \equiv \underbrace{\hat{\beta}_t H(T)\sigma_C^2}_{<0} + \underbrace{\kappa\left(\frac{T-\lambda_G}{\lambda_B\lambda_G}\right)\sigma_M^2}_{<0} < 0, \tag{40}$$

and similarly, $\lim_{t\to 0} G(t) < 0$. This rules out an equilibrium in which brown investors hold all stocks. Moreover, for $t \in [h_U, 1]$,

$$G(t) \equiv \underbrace{\hat{\beta}_t H(T)\sigma_C^2}_{>0} + \underbrace{\kappa\left(\frac{T-\lambda_G}{\lambda_B\lambda_G}\right)\sigma_M^2}_{>0} > 0. \tag{41}$$

Combining (40) and (41), we have that $\forall T \in H_+$, $G(T) \neq 0$, which completes the proof.

Proof of Proposition 3

This proof builds upon Theorem 3.A.5 in Shaked and Shanthikumar (2007). The convex order applied in this theorem is equivalent to a mean-preserving spread. Hence, this theorem states that $\hat{\beta}_j^\dagger$ is a mean-preserving spread of the distribution of $\hat{\beta}_j$ if and only if, $\forall \lambda_G \in [0, 1]$,

$$\int_{\lambda_G}^1 \hat{\beta}_j^\dagger dj > \int_{\lambda_G}^1 \hat{\beta}_j dj \quad \text{and} \quad \int_0^{\lambda_G} \hat{\beta}_j^\dagger dj < \int_0^{\lambda_G} \hat{\beta}_j dj.$$

This immediately implies that:

$$\frac{1}{\lambda_B}\int_{\lambda_G}^1 \hat{\beta}_j^\dagger dj - \frac{1}{\lambda_G}\int_0^{\lambda_G} \hat{\beta}_j^\dagger dj > \frac{1}{\lambda_B}\int_{\lambda_G}^1 \hat{\beta}_j dj - \frac{1}{\lambda_G}\int_0^{\lambda_G} \hat{\beta}_j dj,$$

which completes the proof of sufficiency. To show necessity, note we have that:

$$\mu_\beta = \int_0^1 \hat{\beta}_t dt = \int_0^1 \hat{\beta}_j^\dagger dj.$$

Substituting, we obtain that:

$$\begin{aligned}
\frac{1}{\lambda_B}\int_{\lambda_G}^1 \hat{\beta}_j^\dagger dj - \frac{1}{\lambda_G}\int_0^{\lambda_G} \hat{\beta}_j^\dagger dj &= \frac{1}{\lambda_B}\left(\mu_\beta - \int_0^{\lambda_G} \hat{\beta}_j^\dagger dj\right) - \frac{1}{\lambda_G}\int_0^{\lambda_G} \hat{\beta}_j^\dagger dj \\
&= \frac{1}{\lambda_B}\mu_\beta - \left(\frac{1}{\lambda_B} + \frac{1}{\lambda_G}\right)\int_0^{\lambda_G} \hat{\beta}_j^\dagger dj,
\end{aligned}$$

and the same relation holds upon substituting $\hat{\beta}_t$ for $\hat{\beta}_t^\dagger$ in the above expressions. Therefore,

$$\frac{1}{\lambda_B}\int_{\lambda_G}^{1}\hat{\beta}_j^\dagger dj - \frac{1}{\lambda_G}\int_0^{\lambda_G}\hat{\beta}_j^\dagger dj - \left(\frac{1}{\lambda_B}\int_{\lambda_G}^{1}\hat{\beta}_j dj - \frac{1}{\lambda_G}\int_0^{\lambda_G}\hat{\beta}_j dj\right) > 0 \tag{42}$$

$$\Leftrightarrow \frac{1}{\lambda_B}\mu_\beta - \left(\frac{1}{\lambda_B}+\frac{1}{\lambda_G}\right)\int_0^{\lambda_G}\hat{\beta}_j^\dagger dj - \left(\frac{1}{\lambda_B}\mu_\beta - \left(\frac{1}{\lambda_B}+\frac{1}{\lambda_G}\right)\int_0^{\lambda_G}\hat{\beta}_j dj\right) > 0$$

$$\Leftrightarrow \int_0^{\lambda_G}\hat{\beta}_j^\dagger dj - \int_0^{\lambda_G}\hat{\beta}_j dj < 0.$$

Hence, if inequality (42) holds for all λ_G, Shaked and Shanthikumar (2007)'s Theorem 3.A.5 implies that $\hat{\beta}_j^\dagger$ is a mean-preserving spread of $\hat{\beta}_j$.

Proof of Corollary 3

In this set up, because $\lim_{t\to 0}\hat{\beta}_t = -\infty$, applying Corollary 1, green investors always participate in the market. Therefore, the equilibrium value of T solves the equation:

$$G(T) = 0 \Leftrightarrow \hat{\beta}_T\sigma_C^2 H(T) + \sigma_M^2\left[\frac{\kappa T}{\lambda_G} - \frac{\kappa(1-T)}{\lambda_B}\right] = 0.$$

Recall from the proof of Proposition 2 that $G'(T) > 0$ at the equilibrium value of T, and so:

$$\operatorname{sgn}\left(\frac{\partial T}{\partial \sigma_{\hat{\beta}}}\right) = \operatorname{sgn}\left(-\frac{\partial G}{\partial \sigma_{\hat{\beta}}}\right).$$

Now,

$$\frac{\partial G}{\partial \sigma_{\hat{\beta}}} = \sigma_C^2\left(\frac{\partial \hat{\beta}_T}{\partial \sigma_{\hat{\beta}}}H(T) + \hat{\beta}_T\frac{\partial H}{\partial \sigma_{\hat{\beta}}}\right).$$

From Corollary 2, when $\hat{\beta}_{\lambda_G} > 0$ and $\lambda_G \le \frac{1}{2}$, we have:

$$T < \lambda_G \le \frac{1}{2}. \tag{43}$$

Hence, rewriting $G(T) = 0$ yields:

$$\hat{\beta}_T = -\frac{\kappa\sigma_M^2}{\sigma_C^2 H(T)}\left(\frac{T-\lambda_G}{\lambda_B\lambda_G}\right) > 0.$$

Next, applying the proof of Proposition 3, since increases in $\sigma_{\hat{\beta}}$ generate mean-preserving spreads in the distribution of $\hat{\beta}_j$, we have that:

$$\frac{\partial H}{\partial \sigma_{\hat{\beta}}} = \frac{\kappa}{\lambda_G} \frac{\partial}{\partial \sigma_{\hat{\beta}}} \int_0^T \hat{\beta}_j dj - \frac{\kappa}{\lambda_B} \frac{\partial}{\partial \sigma_{\hat{\beta}}} \int_T^1 \hat{\beta}_j dj < 0.$$

Letting $\Phi(\cdot)$ denote the CDF of a standard normal distribution, we have:

$$\frac{\partial \hat{\beta}_T}{\partial \sigma_{\hat{\beta}}} = \frac{\partial}{\partial \sigma_{\hat{\beta}}} \sigma_{\hat{\beta}} \Phi^{-1}(T) = \Phi^{-1}(T).$$

This has the sign of $T - \frac{1}{2}$, and so, by (43), is negative. Combining these facts, we obtain that:

$$\text{sgn}\left(\frac{\partial T}{\partial \sigma_{\hat{\beta}}}\right) = \text{sgn}\left(-\left[\underbrace{\frac{\partial \hat{\beta}_T}{\partial \sigma_{\hat{\beta}}}}_{<0} \underbrace{H(T)}_{\geq 0} + \underbrace{\hat{\beta}_T}_{>0} \underbrace{\frac{\partial H}{\partial \sigma_{\hat{\beta}}}}_{<0}\right]\right) > 0.$$

Proof of Proposition 4

Because the manager's disclosure decision reduces to disclosing if and only if $\beta_j < \hat{\beta}_{ND}$, the disclosure equilibrium can be solved for by applying the standard arguments from Jung and Kwon (1988). These arguments yield that there is a unique threshold equilibrium in this setting in which the equilibrium disclosure cutoff $\hat{\beta}_{ND}$ solves:

$$\hat{\beta}_{ND} = \frac{(1-p)\,\mu_\beta + p\,(1 - F_\beta(T))\,\mathbb{E}\left[\tilde{\beta}_j | \tilde{\beta}_j \geq \hat{\beta}_{ND}\right]}{(1-p) + p\,(1 - F_\beta(T))},$$

where $F_{\hat{\beta}}(T)$ is the CDF of $\tilde{\beta}_j$. Since this is a weighted average of μ_β and $\mathbb{E}\left[\tilde{\beta}_j | \tilde{\beta}_j \geq \hat{\beta}_{ND}\right] > \mu_\beta$, it exceeds μ_β. Hence, the manager discloses $\tilde{\beta}_j$ whenever $\tilde{\beta}_j \leq \mu_\beta$. Given the assumption that $\tilde{\beta}_{1/2} \leq \mu_\beta$, this implies that firms $j \in \left[0, \frac{1}{2}\right]$ disclose. Consequently, given that $\lambda_G \leq \frac{1}{2}$, we have that:

$$\hat{\beta}_{ND} > \mu_\beta \geq \tilde{\beta}_{1/2} \geq \beta_{\lambda_G}.$$

Taken together, these facts imply that:

$$\int_0^{\lambda_G} \hat{\beta}_j dj = \int_0^{\lambda_G} \beta_j dj. \tag{44}$$

Under the full disclosure regime, we have that non-disclosure is uninformative and results in $\hat{\beta}_{ND} = \mu_\beta \geq \tilde{\beta}_{1/2} = \beta_{\lambda_G}$. Therefore, we again have that equation (9) is satisfied. Finally, note that, given that the efficiency condition can be written as in equation (9), it depends on the distribution of climate betas only through $\int_0^{\lambda_G} \hat{\beta}_j dj$. Since equation (44) holds under both full disclosure and voluntary disclosure, if the equilibrium is efficient under full disclosure, it is also efficient under voluntary disclosure.

Proof of Proposition 5

Denote the upper (lower) bound of the support of $\tilde{\beta}_j$ as $\overline{\beta}$ ($\underline{\beta}$). Because the manager's payoff function given disclosure vs. non-disclosure is always decreasing in β_j, she must disclose only when $\beta_j < \mathcal{T}$, for some threshold $\mathcal{T} \in \left[\underline{\beta}, \overline{\beta}\right]$. The equilibrium condition for such an equilibrium with threshold $\mathcal{T}$, is:

$$m\left(\mathcal{T}\right) = \left(\mu_C - \rho\left(\kappa\mu_\beta + \bar{z}\right)\sigma_C^2\right)\left(\mathcal{T} - \left(1 - F_\beta\left(\mathcal{T}\right)\right)^{-1}\int_{\mathcal{T}}^{\overline{\beta}} x f_\beta\left(x\right) dx\right) - c = 0,$$

where $m\left(\mathcal{T}\right)$ denotes the net benefit to the firm that observes $\beta_j = \mathcal{T}$ to disclosing, and F_β and f_β and the CDF and PDF of $\tilde{\beta}_j$. Lemma 2 in Bagnoli and Bergstrom (2005), together with the fact that $\mu_C - \rho\left(\kappa\mu_\beta + \bar{z}\right)\sigma_C^2 < 0$, implies that $m'\left(\mathcal{T}\right) < 0$. Moreover,

$$\begin{aligned}
&\lim_{\mathcal{T}\to\underline{\beta}} m\left(\mathcal{T}\right) = \left(\mu_C - \rho\left(\kappa\mu_\beta + \bar{z}\right)\sigma_C^2\right)\left(\underline{\beta} - \mu_\beta\right) - c; \\
&\lim_{\mathcal{T}\to\overline{\beta}} m\left(\mathcal{T}\right) = -c < 0.
\end{aligned}$$

Hence, the unique equilibrium when $c > \left(\mu_C - \rho\left(\kappa\mu_\beta + \bar{z}\right)\sigma_C^2\right)\left(\underline{\beta} - \mu_\beta\right)$ involves no disclosure; otherwise, there is a unique equilibrium $\mathcal{T} \in \left(\underline{\beta}, \overline{\beta}\right)$. Now, for $\mathcal{T} \in \left(\underline{\beta}, \overline{\beta}\right)$,

$$\frac{\partial\mathcal{T}}{\partial c} = -\left[\frac{\partial m}{\partial\mathcal{T}}\right]^{-1}\frac{\partial m}{\partial c} < 0,$$

i.e., there is less disclosure as c rises. Moreover, as $c \to 0$, $\lim_{\mathcal{T}\to\overline{\beta}} m\left(\mathcal{T}\right) \to 0$, and so the equilibrium approaches full disclosure. In sum, as disclosure costs grow large, voluntary disclosure becomes completely uninformative, and so, in any equilibrium, $\frac{1}{\lambda_B}\int_{\lambda_G}^1 \hat{\beta}_j dj - \frac{1}{\lambda_G}\int_0^{\lambda_G} \hat{\beta}_j dj \to 0$, i.e., an efficient equilibrium does not exist. In contrast, as $c \to 0$, in any equilibrium, the manager almost always discloses $\tilde{\beta}_j$. Hence, if the equilibrium is efficient under full disclosure, it is also efficient in a voluntary disclosure equilibrium when disclosure costs are low. Moreover, note that $\frac{\partial\mathcal{T}}{\partial c} < 0$, and that, given two disclosure thresholds $\{\mathcal{T}, \mathcal{T}'\}$ with $\mathcal{T}' < \mathcal{T}$, one can reconstruct the information set given $\mathcal{T}$ using the information set given $\mathcal{T}'$. Thus, a decrease in $\mathcal{T}$ renders the disclosure policy more informative in the Blackwell sense, and thus generates a mean-preserving spread in investors' posterior expectations (Baker, 2006). Combining these facts, we have that there is a unique cutoff such that the equilibrium is efficient only when c is below the cutoff.

Proof of Proposition 6

Many of the key steps follow the same structure as the proof of the main results, so I only highlight the essential differences.

Necessity and Sufficiency of Inequality 10 for an Efficient Equilibrium

As in the baseline case, an efficient equilibrium arises if and only if we can construct portfolios $\{D_{Gj}\}_{j\in[0,1]}$ and $\{D_{Bj}\}_{j\in[0,1]}$ that clear the market and satisfy the equations in (22). To assess when this is possible, I find the portfolios that equalize investors' market exposures, satisfy market clearing and the cap on short sales, and minimize the difference between brown and green investors' climate exposures. These portfolios solve:

$$\begin{aligned}
\left\{D^*_{Gj}(\xi)\right\}, \left\{D^*_{Bj}(\xi)\right\} = \underset{\{D_{Gj}\},\{D_{Bj}\}}{\operatorname{argmin}} & \int_0^1 (D_{Bj} - D_{Gj})\, \hat{\beta}_j dj \\
\text{s.t.} & \int D_{Gj} dj = \int D_{Bj} dj = \kappa && \text{(equal market exposure)} \\
& \lambda_B D_{Bj} + \lambda_G D_{Gj} = \kappa && \text{(market clearing)} \\
& D_{Bj} \geq -\xi; D_{Gj} \geq -\xi. && \text{(short-sales cap)}
\end{aligned}$$

Applying the third condition, we can rewrite this in terms of brown investors' portfolio only:

$$\begin{aligned}
\left\{D^*_{Bj}(\xi)\right\} = \underset{\{D_{Bj}\}}{\operatorname{argmin}} & \int_0^1 \left(D_{Bj} - \frac{\kappa - \lambda_B D_{Bj}}{\lambda_G}\right) \hat{\beta}_j dj \\
\text{s.t.} & \int \frac{\kappa - \lambda_B D_{Bj}}{\lambda_G} dj = \int D_{Bj} dj = \kappa && \text{(equal market exposure)} \\
& D_{Bj} \geq -\xi; \frac{\kappa - \lambda_B D_{Bj}}{\lambda_G} \geq -\xi. && \text{(short-sales cap)}
\end{aligned}$$

The associated Kuhn-Tucker conditions are:

$$\begin{aligned}
(D_{Bj} + \xi)\left(\hat{\beta}_j \left(1 + \frac{\lambda_B}{\lambda_G}\right) - \zeta_1 \frac{\lambda_B}{\lambda_G} + \zeta_2\right) &= 0; \\
\left(D_{Bj} - \frac{\kappa + \xi\lambda_G}{\lambda_B}\right)\left(\hat{\beta}_j \left(1 + \frac{\lambda_B}{\lambda_G}\right) - \zeta_1 \frac{\lambda_B}{\lambda_G} + \zeta_2\right) &= 0; \\
\hat{\beta}_j \left(1 + \frac{\lambda_B}{\lambda_G}\right) - \zeta_1 \frac{\lambda_B}{\lambda_G} + \zeta_2 &\leq 0,
\end{aligned}$$

where ζ_1 is the multiplier associated with $\int \frac{\kappa-\lambda_B D_{Bj}}{\lambda_G} dj = \kappa$ and ζ_2 is the multiplier associated with $\int D_{Bj} dj = \kappa$. Note that $\hat{\beta}_j \left(1+\frac{\lambda_B}{\lambda_G}\right) - \zeta_1 \frac{\lambda_B}{\lambda_G} + \zeta_2$ can only be zero at a unique $\hat{\beta}_j$, which implies that, almost everywhere, one of the two short-sales cap constraints will bind. This immediately implies that the portfolios $\left\{D^*_{Gj}(\xi)\right\}$ and $\left\{D^*_{Bj}(\xi)\right\}$ satisfy:

$$D^*_{Gj}(\xi) = \begin{cases} \frac{\kappa(1+\xi)}{\lambda_G} & \text{for } j < T \\ \frac{-\xi\kappa}{\lambda_G} & \text{for } j > T \end{cases} \quad \text{and} \quad D^*_{Bj}(\xi) = \begin{cases} \frac{-\xi\kappa}{\lambda_B} & \text{for } j < T \\ \frac{\kappa(1+\xi)}{\lambda_B} & \text{for } j > T, \end{cases}$$

where T solves the equal market exposures constraint. To solve for T, note that:

$$\int D^*_{Gj}(\xi)\, dj = \frac{\kappa(1+\xi)T}{\lambda_G} - \frac{\kappa\xi(1-T)}{\lambda_G} = \frac{\kappa(T(1+2\xi)-\xi)}{\lambda_G};$$
$$\int D^*_{Bj}(\xi)\, dj = \frac{\kappa(1+\xi)(1-T)}{\lambda_B} - \frac{\kappa\xi T}{\lambda_B} = \frac{\kappa(1+\xi-T(1+2\xi))}{\lambda_B}.$$

Solving for the T that equates these to κ, we obtain $T = \frac{\xi+\lambda_G}{2\xi+1}$. Following identical arguments to those in Lemma 1, we can find portfolios that clear the market and satisfy the equations in (22), and so constitute an efficient equilibrium, if and only if:

$$z_G + \int_0^1 D^*_{Gj}(\xi)\,\hat{\beta}_j dj - \left(z_B + \int_0^1 D^*_{Bj}(\xi)\,\hat{\beta}_j dj\right) \geq 0; \tag{45}$$
$$\Leftrightarrow \left(\frac{\kappa\xi}{\lambda_G} + \frac{\kappa(1+\xi)}{\lambda_B}\right)\int_{\frac{\xi+\lambda_G}{2\xi+1}}^1 \hat{\beta}_j dj - \left(\frac{\kappa(1+\xi)}{\lambda_G} + \frac{\kappa\xi}{\lambda_B}\right)\int_0^{\frac{\xi+\lambda_G}{2\xi+1}} \hat{\beta}_j dj \geq z_G - z_B \geq 0,$$

which verifies the result.

Proof of Subresult (i)

To condense notation, let $T(\xi) \equiv \frac{\xi+\lambda_G}{2\xi+1}$. To prove this result, I show that $\frac{\partial}{\partial\xi}\left(\frac{\kappa(1+\xi)}{\lambda_G}\int_0^{T(\xi)} \hat{\beta}_j dj - \frac{\kappa\xi}{\lambda_G}\int_{T(\xi)}^1 \hat{\beta}_j dj\right) > 0$. Observe that:

$$\frac{\partial}{\partial\xi}\left(\frac{\kappa(1+\xi)}{\lambda_G}\int_0^{T(\xi)} \hat{\beta}_j dj - \frac{\kappa\xi}{\lambda_G}\int_{T(\xi)}^1 \hat{\beta}_j dj\right)$$
$$= \frac{\kappa(1+\xi)}{\lambda_G}\frac{1-2\lambda_G}{(2\xi+1)^2}\hat{\beta}_{T(\xi)} + \frac{\kappa}{\lambda_G}\int_0^{T(\xi)} \hat{\beta}_j dj - \left(-\frac{\kappa\xi}{\lambda_G}\frac{1-2\lambda_G}{(2\xi+1)^2}\hat{\beta}_{T(\xi)} + \frac{\kappa}{\lambda_G}\int_{T(\xi)}^1 \hat{\beta}_j dj\right)$$
$$= \frac{\kappa}{\lambda_G}\left[\frac{1-2\lambda_G}{2\xi+1}\hat{\beta}_{T(\xi)} + \left(\int_0^{T(\xi)} \hat{\beta}_j dj - \int_{T(\xi)}^1 \hat{\beta}_j dj\right)\right]. \tag{46}$$

To complete the proof, I separately consider the cases in which $\lambda_G \leq \frac{1}{2}$ and $\lambda_G > \frac{1}{2}$.

Case 1: $\boldsymbol{\lambda_G \leq \frac{1}{2}}$. By adding and subtracting terms, (46) can be re-written as follows:

$$\begin{aligned}
&\frac{1-2\lambda_G}{2\xi+1}\hat{\beta}_{T(\xi)}+\int_0^{T(\xi)}\hat{\beta}_j dj-\int_{T(\xi)}^1\hat{\beta}_j dj\\
&=\frac{1-2\lambda_G}{2\xi+1}\hat{\beta}_{T(\xi)}+T(\xi)\left[T(\xi)^{-1}\int_0^{T(\xi)}\hat{\beta}_j dj-(1-T(\xi))^{-1}\int_{T(\xi)}^1\hat{\beta}_j dj\right]\\
&\quad+T(\xi)(1-T(\xi))^{-1}\int_{T(\xi)}^1\hat{\beta}_j dj-\int_{T(\xi)}^1\hat{\beta}_j dj\\
&=T(\xi)\left[T(\xi)^{-1}\int_0^{T(\xi)}\hat{\beta}_j dj-(1-T(\xi))^{-1}\int_{T(\xi)}^1\hat{\beta}_j dj\right]+\frac{1-2\lambda_G}{2\xi+1}\hat{\beta}_{T(\xi)}+\left(\frac{2\lambda_G-1}{1-\lambda_G+\xi}\right)\int_{T(\xi)}^1\hat{\beta}_j dj\\
&=T(\xi)\left(\mathbb{E}\left[\hat{\beta}_j|\hat{\beta}_j<T(\xi)\right]-\mathbb{E}\left[\hat{\beta}_j|\hat{\beta}_j>T(\xi)\right]\right)+\frac{1-2\lambda_G}{2\xi+1}\hat{\beta}_{T(\xi)}+\left(\frac{2\lambda_G-1}{1-\lambda_G+\xi}\right)\int_{T(\xi)}^1\hat{\beta}_j dj\\
&<\frac{1-2\lambda_G}{2\xi+1}\hat{\beta}_{T(\xi)}+\left(\frac{2\lambda_G-1}{1-\lambda_G+\xi}\right)\int_{T(\xi)}^1\hat{\beta}_j dj.
\end{aligned}$$

Now, applying the fact that $2\lambda_G-1<0$, we have:

$$\begin{aligned}
\frac{1-2\lambda_G}{2\xi+1}\hat{\beta}_{T(\xi)}+\left(\frac{2\lambda_G-1}{1-\lambda_G+\xi}\right)\int_{T(\xi)}^1\hat{\beta}_j dj &< \frac{1-2\lambda_G}{2\xi+1}+\left(\frac{2\lambda_G-1}{1-\lambda_G+\xi}\right)(1-T(\xi))\hat{\beta}_{T(\xi)}\\
&=\hat{\beta}_{T(\xi)}\left(\frac{1-2\lambda_G}{2\xi+1}+\left(\frac{2\lambda_G-1}{1-\lambda_G+\xi}\right)(1-T(\xi))\right)\\
&=0,
\end{aligned}$$

where the final line follows by substituting for $T(\xi)$ and simplifying.

Case 2: $\boldsymbol{\lambda_G > \frac{1}{2}}$. Again, adding and subtracting terms, (46) can be re-written as follows:

$$\begin{aligned}
&\frac{1-2\lambda_G}{2\xi+1}\hat{\beta}_{T(\xi)}+\int_0^{T(\xi)}\hat{\beta}_j dj-\int_{T(\xi)}^1\hat{\beta}_j dj\\
&=\frac{1-2\lambda_G}{2\xi+1}\hat{\beta}_{T(\xi)}+(1-T(\xi))\left[T(\xi)^{-1}\int_0^{T(\xi)}\hat{\beta}_j dj-(1-T(\xi))^{-1}\int_{T(\xi)}^1\hat{\beta}_j dj\right]\\
&\quad-T(\xi)^{-1}(1-T(\xi))\left(\int_0^{T(\xi)}\hat{\beta}_j dj\right)+\int_0^{T(\xi)}\hat{\beta}_j dj\\
&=(1-T(\xi))\left(\mathbb{E}\left[\hat{\beta}_j|\hat{\beta}_j<T(\xi)\right]-\mathbb{E}\left[\hat{\beta}_j|\hat{\beta}_j>T(\xi)\right]\right)+\frac{2\lambda_G-1}{\xi+\lambda_G}\int_0^{T(\xi)}\hat{\beta}_j dj+\frac{1-2\lambda_G}{2\xi+1}\hat{\beta}_{T(\xi)}\\
&<\frac{2\lambda_G-1}{\xi+\lambda_G}\int_0^{T(\xi)}\hat{\beta}_j dj+\frac{1-2\lambda_G}{2\xi+1}\hat{\beta}_{T(\xi)}.
\end{aligned}$$

Now, since $2\lambda_G - 1 > 0$, we have:

$$\frac{2\lambda_G - 1}{\xi + \lambda_G} \int_0^{T(\xi)} \hat{\beta}_j dj + \frac{1 - 2\lambda_G}{2\xi + 1} \hat{\beta}_{T(\xi)} < \left(\frac{2\lambda_G - 1}{\xi + \lambda_G} T(\xi) + \frac{1 - 2\lambda_G}{2\xi + 1} \right) \hat{\beta}_{T(\xi)} = 0,$$

where the final line again follows by substituting for $T(\xi)$ and simplifying.

Proof of Subresult (ii)

This result follows by essentially identical reasoning to the baseline case. The arguments in Lemmas 2 and 4 show that there must be a T such that, $\forall j < T$, $\Delta_B(j) < 0$ and $\Delta_G(j) = 0$, and $\forall j > T$, $\Delta_G(j) < 0$ and $\Delta_B(j) = 0$. The key change is that, when $\Delta_G(j) = 0$ and $\Delta_B(j) < 0$, the brown investors short sell to the maximum extent possible, setting $D_{Bj} = \frac{\kappa\xi}{\lambda_B}$. Thus, for the market to clear, green investors as a group must hold an aggregate long position of $\kappa(1+\xi)$, corresponding to positions of $\frac{\kappa(1+\xi)}{\lambda_G}$ per investor. Likewise, when $\Delta_G(j) < 0$ and $\Delta_B(j) = 0$, the green investors short sell to the maximum extent possible, setting $D_{Gj} = \frac{\kappa\xi}{\lambda_G}$. Thus, for the market to clear, brown investors as a group must hold an aggregate long position of $\kappa(1+\xi)$, corresponding to positions of $\frac{\kappa(1+\xi)}{\lambda_B}$ per investor.

Proof of Proposition 7

Because the key steps of this proof are similar to the proof of Proposition 1, I leave out several details to focus on the key differences. Investor i's expected utility given a demand function D_{ij} now reduces to:

$$\begin{aligned} EU_i &\propto -\frac{1}{\rho} \mathbb{E}_i \left[\exp\left(-\rho \int D_{ij} \left(\tilde{\alpha}_j + \tilde{\beta}_j (F_C + b_i) + F_M - P_j \right) dj \right) \right] \\ &= -\frac{1}{\rho} \exp \begin{pmatrix} -\rho \left(\int D_{ij} \hat{\beta}_j dj \right) (\mu_C + b_i) + \frac{\rho^2}{2} \left(\int D_{ij} \hat{\beta}_j dj \right)^2 \sigma_C^2 \\ -\rho \left(\int D_{ij} dj \right) \mu_M + \frac{\rho^2}{2} \left(\int D_{ij} dj \right)^2 \sigma_M^2 + \rho \int D_{ij} P_j dj \end{pmatrix} \end{aligned}$$

Now, observe that:

$$\frac{\partial EU_i}{\partial D_{ik}} = \left(\hat{\beta}_k (\mu_C + b_i) + \mu_M - P_k - \rho \hat{\beta}_k \left(\int D_{ij} \hat{\beta}_j dj \right) \sigma_C^2 - \rho \left(\int D_{ij} dj \right) \sigma_M^2 \right) \exp(...).$$

Note this has the sign of:

$$\Delta_i(k) \equiv \hat{\beta}_k (\mu_C + b_i) + \mu_M - P_k - \rho \hat{\beta}_k \left(\int D_{ij} \hat{\beta}_j dj \right) \sigma_C^2 - \rho \left(\int D_{ij} dj \right) \sigma_M^2.$$

Again letting $\Lambda_i \equiv \int D_{ij}\hat{\beta}_j dj$ and $\Omega_i \equiv \int D_{ij} dj$, averaging $\Delta_i(k)$ over investors, and applying market clearing $\sum_{i\in\{G,B\}} \lambda_i \Lambda_i = \kappa\mu_\beta$, we obtain:

$$\int P_j dj = \mu_\beta\left(\mu_C + \bar{b}\right) + \mu_M - \rho\kappa\mu_\beta^2\sigma_C^2 - \rho\kappa\sigma_M^2. \tag{47}$$

Substituting (47) into investor i's first-order condition averaged over firms, we obtain:

$$0 = \mu_\beta\left(b_i - \bar{b} + \rho\sigma_C^2\left(\kappa\mu_\beta - \Lambda_i\right)\right) + \rho\left(\kappa - \Omega_i\right)\sigma_M^2. \tag{48}$$

Following a similar procedure but weighting the investors' first-order conditions in each stock by the stock's expected climate beta $\hat{\beta}_j$ and again integrating over firms, we obtain:

$$\left(\mu_\beta^2 + \sigma_\beta^2\right)\left(b_i - \bar{b} + \rho\sigma_C^2\left(\kappa\mu_\beta - \Lambda_i\right)\right) + \rho\mu_\beta\left(\kappa - \Omega_i\right)\sigma_M^2 = 0. \tag{49}$$

In sum, from equations (48) and (49), we have that $\{\Lambda_i, \Omega_i\}$ must satisfy the following system of equations:we have that $\{\Lambda_i, \Omega_i\}$ must satisfy the following system of equations:

$$\left(\begin{array}{cc} \mu_\beta^2 + \sigma_\beta^2 & \rho\mu_\beta\sigma_M^2 \\ \mu_\beta & \sigma_M^2 \end{array}\right)\left(\begin{array}{c} b_i - \bar{b} + \rho\sigma_C^2\left(\kappa\mu_\beta - \Lambda_i\right) \\ \kappa - \Omega_i \end{array}\right) = \left(\begin{array}{c} 0 \\ 0 \end{array}\right),$$

and the unique solution $\{\Lambda_i, \Omega_i\}$ to this system satisfies:

$$\begin{aligned} \Lambda_i &= \kappa\mu_\beta + \frac{b_i - \bar{b}}{\rho\sigma_C^2}; \\ \Omega_i &= \kappa. \end{aligned} \tag{50}$$

Note that this is effectively identical to the necessary and sufficient conditions for an efficient equilibrium identified in equation (22) upon replacing $z_i - \bar{z}$ by $\frac{b_i - \bar{b}}{\rho\sigma_C^2}$. Thus, we can follow the same set of steps to verify that the condition for an efficient equilibrium to arise is:

$$\frac{1}{\lambda_B}\int_{\lambda_G}^{1} \hat{\beta}_j dj - \frac{1}{\lambda_G}\int_0^{\lambda_G} \hat{\beta}_j dj \geq \frac{b_B - b_G}{\rho\kappa\sigma_C^2}.$$

Proof of Proposition 8

The only change to the equilibrium derivation in this case is in the market-clearing condition in each stock. Specifically, we now have that the demand from indexers plus the demand from the green and brown investors must sum to the per-capita share endowment of κ. Since there is a measure ψ of indexers, this condition reduces to:

$$\begin{aligned}\psi\kappa + (1-\psi)\left(D_{Gj} + D_{Bj}\right) &= \kappa \\ \Leftrightarrow D_{Gj} + D_{Bj} &= \frac{\kappa - \psi\kappa}{1-\psi} = \kappa.\end{aligned}$$

Because this is the same condition as in the baseline analysis, the exact same results hold.

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