The Good Idea Dilemma

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"If it's such a good idea, how come someone else hasn't done it? ... These words of doubt often get so loud that we fail to act." Robert T. Kiyosaki, Rich Dad Poor Dad

Abstract: Once a good idea comes to mind, it is hard to avoid wondering why no one else has already taken advantage of it. Is it because the idea is not as good as it seems? This concern leads to a dilemma of whether to implement or abandon a presumably good idea. The dilemma is tricky – the better the idea seems to be, the more wonder might raise the fact that no one has taken advantage of it. We study the confounded nature of this dilemma in a continuous-time game with multiple players, analyzing the emerging dynamics of idea adoption, and exploring its economic implications. Applying the results to the stock market shows that, in contrast to the efficient market hypothesis, mispricing can persist even when there are many investors and information costs are low.

Keywords: economic opportunities; ideas; information asymmetry; investment; innovation.

1. *Introduction*

People constantly encounter economic opportunities, ranging from minor opportunities such as a used cell phone sold at a bargain price to major opportunities such as technological breakthroughs that can change the world. Opportunities and innovations of all kinds are the main deriving behind economic growth and the creation of welfare (e.g., Schumpeter 1942; Romer 1990). It is therefore difficult to overstate the importance of understanding the incentives of individuals and organizations to take advantage of innovative ideas and opportunities.[1] One potential deterrent to pursuing opportunities and implementing innovative ideas, however, is a natural concern that good ideas inevitably raise. Once a good idea comes to mind, it is hard to avoid questioning why no one else has already implemented it. So, coming across a new promising opportunity often raises the doubt that perhaps the opportunity is not as good as it seems, because otherwise it is likely that someone else would have already taken advantage of it. This concern leads to a dilemma of whether to implement a seemingly good idea or abandon it, which we refer to as "the good idea dilemma". The dilemma arises in numerous situations. Consider, for example, a professor thinking about a low-hanging fruit research idea, wondering why the idea was never published before. Could it be too obvious of an idea to publish? An investor considering the purchase of a stock, wondering why it has been priced below the industry earnings multiple for an extended period. Could it be the result of unfavorable information held by other traders? A technological firm contemplating investing in a new technology, wondering why no other competitor has already developed a similar technology. Could it be a sign that the technology is impractical? A person considering buying a car offered at a bargain price, wondering why the car is on the market for so long. Could it be an indication that the car is damaged? A pedestrian noticing a 20$ bill on the ground, wondering why no one else has picked it up. Could it be that the bill is fake? In all these cases, there seems to be a good opportunity that no one else has already exploited, raising the concern that the opportunity is, in fact, bad. The good idea dilemma is subtle because the very conditions that increase the likelihood of implementing an idea in the absence of the dilemma also hinder implementation by intensifying the concern underlying the dilemma. For example, it is natural to think that more appealing ideas with higher expected gains are more likely to be implemented. It is similarly natural to expect that ideas arriving more frequently (e.g., a low-hanging fruit idea) or opportunities accessible to a broader audience (e.g., a bill on the ground in Fifth Avenue) would have a greater likelihood of being implemented due to their increased accessibility.

[1] In this paper we adopt a broad interpretation of the term 'idea', encompassing a cognitive comprehension of a pathway to create economic value.

However, the more promising and accessible an idea is, the more wonder raises the fact that it has not been successfully exploited before. We aim to study this confounded nature of the good idea dilemma, understand the emerging dynamics of idea adoption, and explore its economic implications.

Our study is based on an analysis of a dynamic game with multiple risk-neutral entrepreneurs (individuals or firms), each of whom may stochastically over time encounter the same opportunity or idea, which provides them access to a risky economic project. In this game, referred to as “the good idea game”, upon receiving access to the project, the entrepreneur needs to decide whether to implement the project or forego it. The entrepreneurs face uncertainty about the return that the project would yield. They believe that the future return from the project is positive in expectations, but upon implementation it may yield a negative return, perhaps due to an initial investment required for implementation. That is, the project seems good on the surface, but upon implementation may turn out to be bad. A key feature of the game is that the project can yield a positive return only when it is implemented for the first time, reflecting the notion that a good idea or opportunity can be exploited only once (e.g., a paper can be published only once, a patent can be registered only once, an offer to purchase a car can only be accepted once, a bill on the ground can be picked up only by one person, etc.). In particular, if the project is indeed good, it yields a positive return upon the first implementation, and after a first successful implementation it disappears or becomes publicly irrelevant (e.g., the paper is published, the patent is registered, the car is not on the market, the bill is no longer on the ground). However, if the project is bad, it can be implemented multiple times, yielding a negative return every time. In this case, the fact that the project is bad remains the private information of the individuals who implement it (e.g., unfruitful research projects do not get published so only researchers trying to publish such a project know it is unpublishable, failed technological endeavors do not materialize to meaningful patents so only technological firms investing in such a technology know it is impractical, only buyers taking a car to car examination know it is in bad condition, only pedestrians picking up the bill from the ground and taking a close look at it know it is fake). As a result, a successful implementation (that yields a positive return) is publicly observable, while unsuccessful implementations (that yield a negative return) are hidden from the public eye. Such an asymmetric flow of information to the market is an inherent feature in many economic settings. This feature is also consistent with the fundamental notion that individuals and firms tend to disclose favorable information and withhold unfavorable information (e.g., Verrecchia 1983; Dye 1985; Nagar 1999; Einhorn 2007; Suijs 2007; Einhorn 2018).

In the unique equilibrium of the good idea game, any entrepreneur receiving access to the project early enough, before a certain endogenous time threshold, chooses to implement it, while any entrepreneur getting access to the project after this time threshold chooses to abandon it. The key intuition for this threshold equilibrium lies in the dynamics of entrepreneurs' joint learning process. Namely, upon receiving access to the project, the entrepreneur knows that the project is still available for implementation either because no one else previously got access to it or because the project is bad and any previous attempts at implementation have resulted in a failure. Over time, the first scenario becomes less probable, and the entrepreneur becomes more concerned that the project is indeed bad. Therefore, as time passes without any past successful implementation of the project, the entrepreneurs decrease their estimate of the expected return from the project upon encountering the opportunity to implement it. As long as their estimate of the expected return from the project is still positive, they choose to implement the project upon receiving the opportunity. At a certain point in time, their estimate of the expected return from the project is reduced to zero. From this point in time onwards, it is no longer beneficial for the entrepreneurs to execute the project. Due to the endogenously growing pessimism about idea quality over time, the time window for exploiting the opportunity is limited by an endogenous time threshold. Beyond this time window, the economy becomes endogenously sterile. Our equilibrium outcome of the limited time window for exploiting an opportunity is reflected in many known economic phenomena. It can explain, for example, why mathematicians abandon old unsolved mathematical problems and a prize is needed to be offered to renew such endeavor (see the Millennium Prize Problems). It could similarly elucidate why the market "dries out" for houses listed for a long time, offering an explanation for the well-known relisting tactic of real estate agents who take off the market houses that do not sell sufficiently fast in order to just put them back on the market again as a new listing (e.g., Smith, Gibler, and Zahirovich-Herbert 2015).

Hence, even though the project yields a positive return in expectations, in equilibrium there is a positive probability that the project would never be implemented by any entrepreneur simply because enough time had passed before it became accessible for the first time. The fundamental properties of the project, as well as the conditions in the economy, obviously affect the probability of implementation. However, beyond their expected direct effect on the implementation probability, they also indirectly affect it by influencing the implementation time threshold set by individuals in response. The overall effect on the probability of implementation depends on the direction and magnitude of the direct and indirect effects, reflecting the nuanced nature of the dilemma. For instance, when considering the ex-ante profitability of the project, the consequent direct and indirect effects on the probability of implementing the project are in the same direction. A more appealing project with higher expected profitability not only directly encourages implementation but also indirectly mitigates concern regarding past failed implementations and thereby causes entrepreneurs to increase the time threshold of implementation, leading to a higher probability of implementation. More confounded are the effects of the number of entrepreneurs in the economy and the frequency of idea arrival on the probability of implementing the project. Naturally, an increase in the number of entrepreneurs that can receive access to the idea has a positive direct effect on the probably of implementation. However, it also indirectly amplifies the entrepreneurs' concern that someone already attempted to implement the idea and failed, leading to a lower time threshold of implementation. Interestingly, it appears that the second indirect effect is so severe that projects that are accessible to more entrepreneurs are less likely to be implemented. This means, for example, that the probability that a bill on the ground in an extremely crowded street will be picked up is very low. In the context of the stock market example, contrary to the standard prediction of the efficient market hypothesis (Fama 1970, 1991), this implies that not only can mispricing persist despite low information costs, but the probability of mispricing persisting also increases as the number of investors in the market rises.[2] Equally surprising is the finding that the probability of implementation is independent of the frequency of its arrival to entrepreneurs. This is because the positive direct effect that an acceleration in the frequency of idea arrival has on the probability of implementing the idea exactly offsets the indirect effect of intensifying the concern regarding the possibility of past failed implementation of the idea, which leads the entrepreneurs to decrease the time threshold of implementation. Hence, counterintuitively, our analysis implies that obvious ideas that are frequently received by people have the same probability of implementation as very creative and complicated ideas. We further show that the dispersion of the project's return distribution plays an important role in determining the implementation probability. In particular, the good idea dilemma causes the entrepreneurs to act as if they are loss-averse, even under the assumption that they are risk-neutral and care only about the expected return. This is because their fear of unobserved failures in the past increases the relative weight that they assign to negative outcomes. Therefore, keeping the expected return from the idea constant, the resulting probability of implementation decreases in the dispersion of the return distribution, even if the entrepreneurs are risk-neutral.

The threshold structure of the entrepreneurs' equilibrium implementation strategy generates welfare inefficiencies. Prior to the equilibrium time threshold for implementation, a bad project with a negative return is implemented multiple times if it has become accessible to more than one entrepreneur early enough.

[2] We further elaborate on the stock market example in the discussion following Proposition 4.

After this time threshold, the economy becomes sterile, implying that a good project with a positive expected return may never be implemented. Hence, in equilibrium, there is over-implementation of bad projects that become accessible to more than one entrepreneur prior to the time threshold of implementation, as well as under-implementation of good projects that become available to the entrepreneurs only after this time threshold. These two inefficiencies coexist in the economy, but there is a trade-off between them, as a higher probability of implementation is associated with a lower probability of under-implementation and a higher probability of over-implementation. The analysis reveals that a higher probability of implementation generally corresponds to a higher level of total welfare in the economy, even though it is also linked to an increased probability of over-implementation. We thus conclude that the main source of welfare distortion that the good idea dilemma generates is under-implementation. While, in general, under-implementation might be of particular concern, because in many cases innovation has positive externalities on society (e.g., a cure for cancer that saves lives, green energy that reduces pollution, etc.), in our setting this is true even ignoring these potential externalities. We further show that, surprisingly, even though individuals set their time threshold for implementing the project to maximize their own expected return, the equilibrium time threshold optimally balances between under and over-implementation in a way that minimizes the value distortion in the entire economy.

In an extension of the analysis, we consider a realistic feature of innovation economics, in which the economic opportunity becomes irrelevant after a certain point in time. This feature captures the notion that in a changing economic environment, the value of economic opportunities may deteriorate over time due to competitive initiatives that arise or innovations that become less relevant as the economy develops. A notable example is the Great Horse-Manure Crisis of 1984 (e.g., Davies 2004), of which ideas for its solution became irrelevant once the automobile was invented shortly after. As in our base analysis, we find that entrepreneurs' implementation strategy is governed by a time threshold that at times can be shorter than the exogenously given expiration date of the opportunity. But, interestingly, we find that total welfare no longer monotonically decreases in the number of competing entrepreneurs. Rather, there is an interior optimal number of entrepreneurs that maximize social welfare. This is because when the opportunity is subject to an expiration date, it is valuable that access to the opportunity would be received early enough in the game. Enhanced competition works for this purpose. We explore how the optimal number of entrepreneurs in the economy depends on the nature of the economic opportunity. We show, in particular, that the optimal number of entrepreneurs in the economy is increasing in the expected profitability of the project. This is because receiving early access to the project is more valuable when the project is expected to yield a higher return. We further show that the optimal number of entrepreneurs in the economy is decreasing in the frequency of the idea arrival. The intuition behind this result is that, in expediting the accessibility of ideas, a higher frequency of idea arrival works as a substitute for a higher number of entrepreneurs. Our results may have policy implications, as they point to the potential impact of government intervention in controlling the size of industries on social welfare.

The good idea game encompasses a wide range of scenarios, as illustrated through various examples, which, to the best of our knowledge, have not been previously explored in the literature. It uniquely combines sequential decision-making by players, unobservability of the game's history, and competition over an opportunity that can be exploited only once if it is good but multiple times if it is bad. The good idea game facilitates learning over time, and it is thus most closely related to the wide literature on learning. The literature on social learning and strategic experimentation (e.g., Bikhchandani, Hirshleifer, and Welch 1992; Banerjee 1992; Rosenberg, Solan and Vieille 2007; Murto and Valimaki 2011; Halac, Kartik and Liu 2017; Margaria 2020) considers games where players learn from others and/or from their own accumulated experience over time. More specifically, the literature on information cascade pertains to situations where players make decisions sequentially, with each observing the actions of those before them. This allows them to infer some of the private information held by previous players before making their own choices. In contrast, in the good idea game, there is no aggregation of personal experience because players face a one-time decision, and learning from others is infeasible due to the invisibility of the sequence of players' arrivals and their actions. Consequently, the learning process of the players in the good idea game relies solely on the passage of time. Furthermore, a good idea can be exploited only once, leading to competition among players. The lack of observability, combined with the competitive nature of the game, results in the interesting characteristic that a higher probability of implementation by all players reduces the incentive of each player to implement, creating the confounded nature of the game. This leads to unexpected results, such as the decrease in social welfare and the likelihood of a good idea being implemented as the number of players in the game increases.

Although the good idea dilemma arises in a wide scope of economic situations that includes all kinds of paths for creating economic value, it also pertains to the important special case of creating value by implementing innovative ideas, and therefore relates to the innovation literature as well. We depart from the innovation literature, which mostly focuses on the investment in technological breakthrough arrival and often do not distinguish it from the subsequent stage of implementation (e.g., Grossman and Shapiro 1987; Loury 1979; Lee and Wilde 1980), by exploring the dynamics in the late stage of implementing ideas following their arrival. While some studies in the innovation literature explicitly consider the implementation stage (e.g., Feng, Luo, Michaeli 2024), they focus on different aspects of this stage, such as issues of delegation and incentive contracting. The good idea dilemma also contributes to the asymmetric information literature, where the less informed party responds to their informational disadvantage by exercising caution, often resulting in market inefficiencies. For example, in the adverse selection theory of Akerlof (1970), the buyer of an asset, being less informed about the asset's true value than the seller, becomes skeptical about the asset's value and is willing to purchase the asset only at a very low price. This skepticism leads to market failure, as high-quality assets are driven out of the market. Similarly, in the winner's curse described by Capen, Clapp and Campbell (1971), the absence of higher bids in an auction leads the winning bidder to question the true worth of the item. To protect themselves from overpaying, bidders tend to place discounted bids, which can result in the item being sold for less than its true value, introducing inefficiency into the auction process. In the good idea dilemma, the fact that an opportunity is still available, suggesting that others may have unsuccessfully tried to exploit it, raises doubts about its value. In response, players set a limited time window for implementation, resulting in both under- and over- implementations of opportunities.

The paper proceeds as follows. Section 2 describes the good idea game underlying the analysis. In Section 3, we establish the existence and uniqueness of the equilibrium in our game, characterize its structure, and analyze its properties. In Section 4, we extend our analysis by introducing into the base game an expiration date for the project. The final section summarizes and offers concluding remarks. Proofs appear in the appendix.

2. The Good Idea Game

Our analysis is based on a winner-take-all innovation game with $n \geq 2$ risk-neutral entrepreneurs (individuals or firms). All the assumptions and parameters underlying the game are assumed to be commonly known to all players, unless otherwise indicated. The game (henceforth, the good idea game) pertains to one particular idea for an economic initiative that each of the n entrepreneurs randomly and privately receive at some point in time $t \geq 0$, where $t = 0$ represents the earliest time at which the idea is feasible. For any $i \in \{1,2,\dots,n\}$, the random variable $\tilde{t}_i$ represents the point in time at which entrepreneur i receives the idea. It is assumed that the random variables $\tilde{t}_1, \tilde{t}_2, \dots, \tilde{t}_n$ are independent and identically distributed over the support $[0,\infty)$ with a continuous probability distribution function f and a cumulative distribution function F, satisfying $F(0) = 0$ and $\lim_{t\to\infty} F(t) = 1$.[3] The idea is modeled as a risky project that yields an uncertain return.[4] An entrepreneur receiving the idea needs to decide whether to execute the project and receive the uncertain return or forego it. Differently from most studies in the innovation literature, which focus on the investment in technological breakthrough arrival and often do not distinguish it from the subsequent stage of implementation (e.g., Grossman and Shapiro 1987; Loury 1979; Lee and Wilde 1980), we aim at understanding the implementation of innovative ideas following their arrival. To facilitate the analysis, we abstract from the up-front investment in technological breakthrough arrival by adopting instead the simplifying assumption that the arrival of the innovative opportunity is exogenous.

We assume that the implementation of the project by an entrepreneur is not observable to other entrepreneurs. The first entrepreneur implementing the project receives a random return $\tilde{r}$, which takes the positive value $r_H > 0$ with probability q and the negative value $r_L < 0$ with probability $1 - q$. Accordingly, the expected return $E[\tilde{r}]$ from the first implementation of the project equals $qr_H + (1 - q)r_L$ and is denoted $\bar{r}$. We assume that $\bar{r}$ is positive, so that a priori the project is profitable.[5] We define success in carrying out the project as a scenario where an entrepreneur receives access to the project, decides to implement the project, and the project yields a positive return r_H to the entrepreneur. Similarly, we define failure in executing the project as a scenario where an entrepreneur receives access to the project, decides to implement it, and the project yields a negative r_L return to the entrepreneur. Importantly, the project can yield a positive return r_H only once, and only for the first entrepreneur who implements it.[6] After a first profitable implementation of the project, the project disappears or becomes publicly irrelevant, and the game is over. This reflects the notion that a good idea or initiative opportunity can be exploited only once. However, if the

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[3] The assumption $\lim_{t\to\infty} F(t) = 1$ implies that each entrepreneur necessarily receives an access to the project at some point in time. We will relax this assumption in section 4.

[4] We use the terms idea, opportunity, and project interchangeably throughout the paper.

[5] In general, the project's return may deteriorate over time (e.g., due to competitive ideas that arise as time passes or innovations that become less relevant as the environment develops). We abstract from this possible effect in order to isolate a different, so far unexplored, effect that also works to endogenously decrease the value of the project over time. We will introduce exogenous deterioration in the project's return in section 4.

[6] Due to the continuous time assumed in our dynamic game, the probability that more than one entrepreneur will receive an access to the project at any given point of time is negligible. We thus do not need to explicitly model the way the return from the project is split between entrepreneurs who implement the project simultaneously. For completeness, it can be assumed that if the first implementation is made by more than one entrepreneur, any positive return from the project is equally split between the entrepreneurs who implement it, or alternatively, that one entrepreneur is randomly chosen to receive the entire return of the project.

first implementation of the project yields a negative return r_L (i.e., resulted in a failure), it can still be potentially implemented multiple times by different entrepreneurs, yielding the negative return r_L every time. In this case, the failure in carrying out the project is privately known only to the entrepreneurs who implemented it.

We represent the implementation strategy of each entrepreneur $i \in \{1,2,\dots,n\}$ upon receiving access to the project by the function $I_i: [0,\infty) \to \{0,1\}$. Specifically, $I_i(t) \in \{0,1\}$ is the binary implementation decision of entrepreneur i at time $t \in [0,\infty)$ upon receiving access to the project (i.e., t is the realization of the random variable $\tilde{t}_i$).[7] The decision to implement the project is represented by $I_i(t) = 1$, whereas the decision to forego the project is depicted by $I_i(t) = 0$.[8] Equilibrium in the good idea game is formally defined as a vector $(I_1, I_2, \dots, I_n: [0,\infty) \to \{0,1\})$, which includes the implementation strategies of all n entrepreneurs. We look for Subgame Perfect Nash equilibria, in which at any point in time the entrepreneurs make optimal implementation decisions that maximize their expected utility based on their available information and their rational expectations about the strategic behavior of all other entrepreneurs, utilizing Bayes' rule to make inferences and update their beliefs.

3. *Equilibrium Analysis*

In equilibrium of the good idea game, each entrepreneur $i \in \{1,2,\dots,n\}$, who received the idea at time $t \in [0,\infty)$, chooses to implement the idea by executing the project if and only if the entrepreneur assesses that the expected return from implementing the project at time t is positive. In forming and updating the expectations about the return from the project, the entrepreneur takes into account that the absence of a successful implementation of the project up to time t may be the result of two possible scenarios. The first is that at least one other entrepreneur has already received access to the project and implemented it, but the implementation failed. The second possible scenario is that no other entrepreneur has already implemented the project, either because no other entrepreneur got access to the project prior to time t or because one or more entrepreneurs got access to the project before time t but decided not to execute it. In the former scenario, the expected return from implementing the project equals r_L and it is thus negative. In the latter scenario, the project was never implemented before, so the expected return

[7] The assumption that entrepreneurs make their implementation decision upon receiving an access to the project is without loss of generality. As will be clear later, delaying the implementation of the project to a later time is irrational because it decreases the probability of success.

[8] We focus on equilibria with pure strategies, but allowing for mixed strategies would not change the results.

from implementing the project equals $\bar{r}$ and it is thus positive. For any $i \in \{1,2,\ldots,n\}$, we denote by $\pi_i(t)$ the Bayesian consistent probability that entrepreneur i rationally ascribed at time t to the first scenario upon receiving the accesses to the project. Using this notation, it follows that entrepreneur i estimates that the expected return from the project implementation at time t is $\pi_i(t)r_L + \left(1-\pi_i(t)\right)\bar{r}$. Accordingly, entrepreneur i chooses to implement the project at time t if $\pi_i(t)r_L + \left(1-\pi_i(t)\right)\bar{r} > 0$ and to abandon it otherwise. In deriving the equilibrium in the game, we therefore need to explore the characteristics of the probability $\pi_i(t)$, which are described in the following lemma.

LEMMA 1 (BELIEFS UPDATING OVER TIME). *For any* $i \in \{1,2,\ldots,n\}$ *and* $t \in [0,\infty)$, *the probability* $\pi_i(t)$ *equals* $\pi_i(t) = \frac{(1-q)\theta_i(t)}{1-q\theta_i(t)}$, *where* $\theta_i(t) \equiv 1 - \prod_{j\neq i}(1-\int_0^t I_j(k)f(k)dk)$. *The probability* $\pi_i(t)$ *is a continuous and weakly increasing function of* t, *where* $\pi_i(0) = 0$.

The probability $\pi_i(t)$ depicts a rational doubt that comes to the mind of entrepreneur i when receiving the innovative idea at time t. This is the doubt that arises from questioning why no one else has yet taken advantage of this opportunity. Lemma 1 indicates that this doubt is increasing over time. Intuitively, as time passes, it becomes increasingly likely that at least one of the entrepreneurs has already received the idea and gained access to the project. So, when moving forward in time, the fact that the opportunity has not yet been exploited becomes more likely to be the result of a failed past implementation of the project rather than the consequence of the absence of previous implementations of the project. Given that the probability $\pi_i(t)$ of a past failure is increasing in the time t, the entrepreneur's estimate of the expected return from the project, $\pi_i(t)r_L + \left(1-\pi_i(t)\right)\bar{r}$, is decreasing in t. It follows that the incentive of the entrepreneur to implement the project is diminishing over time. This implies that it is economically beneficial to the entrepreneurs to implement the project if and only if they receive access to the project early enough. Based on this observation, the equilibrium in the game is derived. The following proposition establishes the existence and uniqueness of equilibrium in the good idea game and characterizes its structure.

PROPOSITION 2 (EQUILIBRIUM). *There exists a unique equilibrium* $(I_1, I_2, \ldots, I_n: [0,\infty) \to \{0,1\})$ *in the game. The equilibrium is symmetric and characterized by a positive time threshold* $t^* > 0$, *satisfying:* $I_i(t) = \begin{cases} 1 \ if \ t < t^* \\ 0 \ otherwise \end{cases}$ *for any* $i \in \{1,2,\ldots,n\}$ *and* $t \in [0,\infty)$. *The equilibrium time threshold* t^* *is the unique solution of the equation* $F(t^*) = 1 - R^{\frac{1}{n-1}}$, *where* $R = -\frac{(1-q)r_L}{qr_H}$. *In equilibrium,* $\pi_i(t) =$

$\begin{cases} \pi(t) & if\ t \in [0, t^*] \\ \pi(t^*) & if\ t \in (t^*, \infty) \end{cases}$ *for any* $i \in \{1,2,\dots,n\}$ *and* $t \in [0,\infty)$, *where* $\pi(t) = \frac{(1-q)\left(1-(1-F(t))^{n-1}\right)}{1-q\left(1-(1-F(t))^{n-1}\right)}$ *and* $\pi(t^*)r_L + \left(1-\pi(t^*)\right)\bar{r} = 0$.

Proposition 2 indicates that the unique equilibrium in the good idea game is a symmetric equilibrium, in which all the entrepreneurs adopt an identical strategy. According to this strategy, each entrepreneur chooses to implement the project if and only if the time of receiving access to the project is sufficiently early, before a certain time threshold t^*. That is, the time window for exploiting the opportunity is limited by the endogenous time threshold t^*. Prior to the time threshold t^*, every entrepreneur receiving access to the project chooses to implement it. But, from time t^* onwards, it is no longer beneficial to the entrepreneurs to implement the project and the economy endogenously becomes sterile. This shape of the equilibrium follows from Lemma 1, and it is the consequence of the escalation over time in the probability $\pi_i(t)$ that each entrepreneur $i \in \{1,2,\dots,n\}$, who receives that access to the project, assigns to the scenario that the project has been already implemented and failed.

In equilibrium, the functions $\pi_1(t), \pi_2(t), \dots, \pi_n(t)$ coincide with the increasing function $\pi(t)$ for any time t preceding the time threshold t^*, and equal the constant $\pi(t^*)$ for any time t following the time threshold t^*. At time $t = 0$, the probability $\pi(0)$ is zero, and the expected return from the project equals $\bar{r}$ and it is thus positive. So, all the risk-neutral entrepreneurs rationally choose to execute the risky project at time $t = 0$. As the time t passes without any successful implementation of the project, the probability $\pi(t)$ increases, and the expected return $\pi(t)r_L + \left(1-\pi(t)\right)\bar{r}$ from the project decreases. As long as the expected return $\pi(t)r_L + \left(1-\pi(t)\right)\bar{r}$ from the project is still positive, the entrepreneurs rationally choose to execute the project. At a certain point in time, which is the equilibrium time threshold t^*, the expected return from the project $\pi(t^*)r_L + \left(1-\pi(t^*)\right)\bar{r}$ becomes so low that it equals zero. Hence, at the time threshold t^*, the project becomes not profitable. Accordingly, after the time threshold t^*, it is no longer beneficial for the entrepreneurs to execute the project. As a result, the equilibrium time threshold t^* for implementing the project is the solution of the equation $\pi(t^*)r_L + \left(1-\pi(t^*)\right)\bar{r} = 0$, which is equivalent to the equation $F(t^*) = 1 - R^{\frac{1}{n-1}}$ by Proposition 2. The ratio $R = \frac{-(1-q)r_L}{qr_H}$ on the right-hand side of the equilibrium equation summarizes the economic properties of the project into a single measure, which varies between zero and one because $r_L < 0$, $r_H > 0$, $q \in (0,1)$, and $\bar{r} = qr_H + (1-q)r_L > 0$. It follows from $R \in (0,1)$ and $n \geq 2$ that $R^{\frac{1}{n-1}} \in (0,1)$, implying that the right-hand side of the equilibrium equation is a constant that belongs to the interval $(0,1)$. The function $F(t^*)$ on the left-hand side of the equilibrium equation is the cumulative distribution function of the random variables $\tilde{t}_1, \tilde{t}_2, \ldots, \tilde{t}_n$, and it is thus continuous and increasing in t^*, where $F(0) = 0$ and $\lim_{t^* \to \infty} F(t^*) = 1$. Therefore, the equilibrium equation $F(t^*) = 1 - R^{\frac{1}{n-1}}$ has a unique solution, establishing the existence and uniqueness of the equilibrium in the game. The resulting equilibrium time threshold t^* is critical in determining the economic implications of the good idea dilemma. Proposition 3 describes the sensitivity of the equilibrium time threshold t^* to the modeling parameters.

PROPOSITION 3 (TIME THRESHOLD). *The equilibrium time threshold* t^* *is increasing in* r_H, *increasing in* r_L, *increasing in* q, *and decreasing in* n. *For any two cumulative distribution functions* F_1 *and* F_2 *over the support* $[0, \infty)$, *satisfying* $\forall t > 0: F_1(t) < F_2(t)$, *the equilibrium time threshold* t^* *under* $F = F_1$ *is higher than the equilibrium time threshold* t^* *under* $F = F_2$.

The equilibrium time threshold t^* is the point in time at which the expected return from the project changes from a positive value to zero. Therefore, any change in the modeling parameters that works to increase (decrease) the expected return from the project also leads to a consequent shift of the equilibrium time threshold t^* further into the future (backward into the past). In particular, at any time $t \in [0, \infty)$, the expected return $\pi(t)r_L + (1 - \pi(t))\bar{r}$ from the project is clearly increasing in both r_L and r_H, and so is the time threshold t^*. Similarly, at any time t, the expected return $\pi(t)r_L + (1 - \pi(t))\bar{r}$ from the project is increasing in the probability q of success in the project. This is not only because $\bar{r}$ is clearly increasing in the success probability q, but also because the probability $\pi(t)$ of a past failure of the project is decreasing in q. Consequently, the time threshold t^* is increasing in q. The sensitivity of the time threshold t^* to the characteristics of the project, as reflected by the parameters r_L, r_H and q, implies that the time threshold is higher for better ideas, which provide access to more profitable projects.[9] The time threshold, however, depends not only on the economic properties of the project, but also on the level of competition in the market, as captured in the model by the number n of entrepreneurs. As the number

[9] It should be however noted that the project's profitability affects the equilibrium time threshold t^* only via the ratio $R = \frac{-(1-q)r_L}{qr_H}$, as implied by the equivalence of the equilibrium equation $\pi(t^*)r_L + (1 - \pi(t^*))\bar{r} = 0$ to the equation $F(t^*) = 1 - R^{\frac{1}{n-1}}$. While both measures $\bar{r} = qr_H + (1-q)r_L$ and $R = \frac{-(1-q)r_L}{qr_H}$ capture the profitability of the project, they are not identical. In particular, any change in the parameters r_H, r_L and q that increases (decreases) the profitability $\bar{r} = qr_H + (1-q)r_L$ of the project also increases (decreases) the ratio $R = \frac{-(1-q)r_L}{qr_H}$, but simultaneous changes in r_H, r_L and q that do not affect one of the two measures may nevertheless alter the other measure.

n of entrepreneurs increases, so does the probability that at least one of the n entrepreneurs has already received access to the project and implemented it in the past. Therefore, at any time t, the probability $\pi(t)$ is increasing in n, implying that the expected return $\pi(t)r_L + (1 - \pi(t))\bar{r}$ from the project is decreasing in n, and so is the time threshold t^*. Another determinant of the probability $\pi(t)$ is the shape of the cumulative distribution function F of the random variables $\tilde{t}_1, \tilde{t}_2, \dots, \tilde{t}_n$. As the function F attains a higher value for any $t \in [0, \infty)$, the access to the project is more likely to arrive earlier to the entrepreneurs, and so the probability of a past implementation of the project becomes higher, increasing the probability $\pi(t)$ and decreasing in response the time threshold t^*.[10] The sensitivity of the equilibrium time threshold t^* to the parameter n and to the shape of the function F suggests that the time threshold is lower for ideas that are accessible to more entrepreneurs and more frequently observed by them. It follows that an entrepreneur receiving access to the project at time $t \in (0, \infty)$ will choose to exploit it if the project is sufficiently profitable (that is, if r_L, r_H or q are sufficiently high), or if n or $F(t)$ are sufficiently low.

In equilibrium, there is over-implementation of bad projects that become accessible to more than one entrepreneur before the implementation time threshold t^*, as well as under-implementation of good projects that become available for the first time only after the time threshold t^*. Under-implementation is defined as a scenario in which the project is never implemented, even though its ex-ante expected return is positive. Over-implementation is defined as a scenario in which a bad project that yields a negative return r_L is implemented by more than one entrepreneur. Under-implementation is of particular concern, because in many cases innovation has positive externalities on society, beyond what is directly reflected in the return to the individual entrepreneur (e.g., a cure for cancer that saves lives, green energy that reduces pollution, etc.). It is therefore interesting to understand the circumstances in which under-implementation is more likely to occur. Proposition 4 characterizes the probability of under-implementation and presents its sensitivity to changes in the modeling parameters.

[10] Consider, for example, the special case of $\tilde{t}_1, \tilde{t}_2, \dots, \tilde{t}_n \sim Exp(\lambda)$, where the time of idea arrival has an exponential distribution with $\lambda > 0$. In this case, the cumulative distribution function is $F(t) = 1 - exp(-\lambda t)$. The parameter λ captures the idea arrival rate. The higher is λ, the higher is the probability $F(t)$ of idea arrival up to any given time $t \in [0, \infty)$. Accordingly, as the idea arrival rate λ increases, the equilibrium time threshold t^* decreases.

PROPOSITION 4 (UNDER-IMPLEMENTATION). *In equilibrium, the probability of under-implementation is* $\left(1-F(t^*)\right)^n$*. It is positive, decreasing in* r_H*, decreasing in* r_L*, decreasing in* q*, increasing in* n*, and independent of* F.

Proposition 4 indicates that the probability of under-implementation is $\left(1-F(t^*)\right)^n$. Intuitively, the probability of under-implementation under the equilibrium presented in Proposition 2 is equal to the probability that no entrepreneur will receive access to the project by the time threshold t^* of implementing the project. For any $i \in \{1,2,\dots,n\}$, $Pr(\tilde{t}_i > t^*) = 1 - F(t^*)$ is the probability that entrepreneur i will receive the access to the project after the time threshold t^*, and therefore $\prod_{i=1}^{n} Pr(\tilde{t}_i > t^*) = \left(1-F(t^*)\right)^n$ is the probability that none of the n entrepreneurs will receive access to the project up to the time threshold t^*. Proposition 4 further suggests that the probability of under-implementation is decreasing in the profitability parameters r_H, r_L and q, implying that appealing projects are less exposed to under-implementation. This is because the equilibrium time threshold t^* is increasing in the profitability parameters, as indicated by Proposition 3.

The effect of the number n of entrepreneurs on the probability of under-implementation is subtler. An increase in n has a direct effect of decreasing the probability of under-implementation due to the increased probability that the project will be observed and executed by at least one entrepreneur up to any given time threshold. However, this increased probability of implementation creates a countervailing indirect effect of increasing the probability of under-implementation because it exacerbates the concern regarding the possibility of past failed implementation and thereby decreases the time threshold t^*, as shown in Proposition 3. Surprisingly, it follows from Proposition 4 that even though the indirect effect occurs in response to the direct effect, the indirect effect dominates, so that the probability of under-implementation is counterintuitively increasing in n. This means, for example, that the probability of selling a car when there are many potential buyers in the market is relatively low, and that the probability that a bill on the ground will be picked up in an extremely crowded street is very low. Interestingly, in the context of investment in competitive markets, unlike the prediction from the existing literature that competition might encourage early investment or over-investment (e.g., Loury 1979; Lee and Wilde 1980; Reinganum 1981; Fudenberg, Gilbert, Stiglitz, and Tirole 1983; Fudenberg and Tirole 1985; Harris and Vickers 1985, 1987; and Grossman and Shapiro 1987), our analysis shows that competition might rather cause firms to forego investment.

The results of Proposition 4 can provide interesting insights also in the context of the stock market example. This example, however, requires some elaboration. Consider a stock that is priced below a standard benchmark based on public information, such as industry specific earnings or book value multiple. Let time $t = 0$ represent the moment when the stock price falls below this benchmark. As time passes, investors randomly notice that the stock is underpriced relative to the benchmark. They then must decide whether to gather more information about the stock (such as learning about the underlying firm, understanding its operations, analyzing its relative strengths and weaknesses, and reading its financial statements, press releases, and analysts' reports) at a cost $c > 0$ in order to determine if the price drop indicates a genuine underpricing (a probability q event) or if it reflects unfavorable public information about the firm's growth rate (a probability $1-q$ event). If the stock is indeed underpriced, the investor can profit by buying the stocks offered at the discounted price, yielding a profit $v > 0$.[11] Letting $r_H = v - c$ and $r_L = -c$, the ex-ante return for gathering information is $\overline{r} = qr_H + (1-q)r_L$. Assuming that the information cost is sufficiently low that $\overline{r} > 0$, this example is nested within the good idea game. Ignoring the dilemma raised in this paper, the efficient market hypothesis of Fama (1970, 1991) predicts that the first investor who observes the underpricing relative to the benchmark would gather information, and if the underpricing is real, he would act on it, thereby eliminating the underpricing. As the number of investors in the market becomes very large, this adjustment should happen almost instantly. The good idea dilemma, however, suggests otherwise. It predicts that with a positive probability, no investor who observes the apparent underpricing will gather information and eliminate it. Moreover, this probability increases as the number of investors in the market rises. This highlights the notion that even with many investors and low information costs, stock underpricing can persist.

The idea arrival process also has two countervailing effects on the probability of under-implementation. On the one hand, a higher frequency of idea arrival, as captured in the model by a higher $F(t)$ for every t, has a direct effect of decreasing the probability of under-implementation due to the increase in the probability of the entrepreneurs to receive access to the project early on. On the other hand, a higher frequency of idea arrival also has an indirect effect of decreasing the probability of under-implementation because it intensifies the concern regarding the possibility of past failed implementation of the project and thereby decreases the time threshold t^*, as shown in Proposition 3. Interestingly, Proposition 4 indicates that these two opposite forces exactly offset each other. While an acceleration in

[11] The example equally applies to overpricing if short selling is allowed.

the frequency of idea arrival, as captured by a higher $F(t)$ for every t, increases the probability of receiving an access to the project by time t^*, in equilibrium, the time threshold t^* is proportionally reduced to fully offset the change in F. Consequently, as reflected in Proposition 4, the probability of under-implementation is independent of the function F. This means that straightforward ideas that arrive very frequently (e.g., low hanging fruit research ideas or obvious solutions to common problems) have the same ex-ante probability of being implemented (and thus the same probability of under-implementation) as very creative ideas that arrive rarely (e.g., out-of-the-box solutions to complex problems). This is because, while straightforward ideas are very likely to be observed early in time, once observed they are less likely to be implemented because the dilemma in these cases is severe and in response the entrepreneurs set the time threshold of implementation very low. In contrast, creative ideas are not likely to be observed early, but once observed they are very likely to be implemented because the concern that the dilemma raises in these cases is mild and the endogenous time threshold of implementation is high.

In conclusion, the probability of under-implementation is decreasing in the profitability of the project (as captured in the model by the parameters r_H, r_L and q), increasing in the number of entrepreneurs (as captured by the parameter n), and independent in the frequency of idea arrival (as captured by the function F). It might be expected that conditions that work to increase (decrease) the probability of under-implementation would have the opposite effect of decreasing (increasing) the probability of over-implementation. Indeed, the analysis reveals that in most cases, albeit not always, there is a trade-off between the probability of under and over-implementation. To explore this trade-off, we characterize the probability of over-implementation and present its sensitivity to changes in the modeling parameters in Proposition 5.

PROPOSITION 5 (OVER-IMPLEMENTATION). *In equilibrium, the probability of over-implementation is* $(1-q)\sum_{m=2}^{n}\frac{n!}{m!(n-m)!}F(t^*)^m\big(1-F(t^*)\big)^{n-m}$. *It is positive, increasing in r_H, increasing in r_L, non-monotonic in q (initially increasing in q, attaining a maximum, and then decreasing in q), decreasing in n, and independent of F.*

Proposition 5 shows that to a large extent, changes in the modeling parameters that lead to more (less) under-implementation, also lead to less (more) over-implementation. This is because conditions that generate a lower (higher) equilibrium time threshold t^* frequently work to increase (decrease) the probability of under-implementation while decreasing (increasing) the probability of over-implementation. The only exception in our setting is the effect of changes in the parameter q on the probabilities of under-implementation and over-implementation. While Proposition 4 indicates that the probability of under-implementation is decreasing in q, it follows from Proposition 5 that the probability of over-implementation is non-monotonic in q. To understand the non-monotonicity of the probability of over-implementation with respect to the parameter q, it is convenient to first consider the edge cases of extremely high value of q and extremely low value of q. When the parameter q is very high, approaching 1, implementation is likely to lead to success and failures rarely occur, so over-implementation almost never occurs. On the other extreme, when q is very low, approaching 0, implementation is likely to lead to a failure, and thus the entrepreneurs have less incentive to implement the project. In this case, the entrepreneurs set the time threshold t^* very low, and as a result, implementation (and failure) occurs with low probability, so again over-implementation almost never occurs. This suggests that over-implementation is more of a concern when the probability q of a successful project is moderate.

While both inefficiencies of under-implementation and over-implementation generate value distortion, it is unclear how they interactively affect the overall expected return in the entire economy, because generally the parameters of the model have opposite effects on the probability of under-implementation and the probability of over-implementation. We refer to the overall expected return in the entire economy as the welfare in the economy and denote it $W(t^*)$, reflecting its dependence on the equilibrium implementation time threshold t^*. Proposition 6 explores the determinants of the ex-ante welfare $W(t^*)$ in the economy and demonstrates that it is always strictly lower than the first-best welfare of $\bar{r} = qr_H + (1-q)r_L$, which is the expected return that the project yields when it is implemented only once by a single entrepreneur.

PROPOSITION 6 (WELFARE). *In equilibrium, the ex-ante welfare of the entire economy is given by* $W(t^*) = \sum_{m=1}^{n} \frac{n!}{m!(n-m)!} F(t^*)^m \big(1 - F(t^*)\big)^{n-m} (qr_H + m(1-q)r_L)$ *and it is strictly lower than the first-best welfare* $\bar{r} = qr_H + (1-q)r_L$*. The welfare* $W(t^*)$ *is increasing in* r_H*, increasing in* r_L*, increasing in* q *, decreasing in* n*, and independent of* F*.*

Proposition 6 shows that the good idea dilemma leads to welfare distortion in the economy. Specifically, the ex-post expected return in the entire economy is strictly lower than the first-best return of $\bar{r} = qr_H + (1-q)r_L$, which is obtained when the project is implemented only once by a single entrepreneur. The welfare distortion stems both from under-implementation and over-implementation of the project. As both the probability of under-implementation and the probability of over-implementation are independent of the function F, so is the welfare of the economy. Hence, the welfare is not affected by the frequency of idea arrival, as captured by the function F. Most of the other modeling parameters have opposite effects on the probability of under-implementation and the probability of over-implementation, but it appears that the countervailing effects do not exactly offset each other. Proposition 6 rather suggests that the effect of changes in the parameters r_H, r_L, q and n on the probability of under-implementation dominates their effect on the probability of over-implementation. That is, welfare is positively related to profitability parameters r_H, r_L and q, but negatively related to the number n of entrepreneurs in the economy.

[Figure 1]

Figure 1 summarizes graphically the results of Propositions 4, 5, and 6. In all four plots, three equilibrium outcomes are illustrated: the probability of under-implementation in green, the probability of over-implementation in blue, and the welfare in red. These three equilibrium outcomes are described as functions of the parameter r_H in the top left plot, as functions of the parameter r_L in the top right plot, as functions of the parameter q in the bottom left plot, and as functions of the parameter n in the bottom right plot. Except for the bottom left plot that pertains to the parameter q, in all other plots the green curve and the blue curve move in opposite directions, demonstrating a trade-off between the probability of under-implementation and the probability of over-implementation. Also, in all four plots, the red curve and the green curve move in opposite directions, demonstrating that the effect of under-implementation on the welfare distortion dominates the effect of over-implementation. This implies that, even though both under-implementation and over-implementation cause welfare distortion, the main source of welfare distortion is under-implementation. This is even in a model that precludes the positive externalities that innovations frequently have on society. This result is surprising in light of the well-established argument that the aggregate investment in research and development in the economy is too high relative to the cooperative optimum because of the race between entrepreneurs to create and patent an invention (e.g., Reinganum 1981). While the innovation literature points to overinvestment in the earlier stage of seeking an innovative idea, our study indicates that under-implantation is the main cause of welfare distortion subsequent to the idea arrival.

The equilibrium time threshold t^* clearly influences the welfare of the economy due to its effect on both the probability of under-implementation and the probability of over-implementation. The time threshold is chosen in equilibrium by each entrepreneur on purpose to maximize his own expected profit, ignoring the effect that it has on the other entrepreneurs. Nevertheless, Proposition 7 shows that the equilibrium time threshold t^* maximizes the total expected return in the economy. It follows from Proposition 7 that even though entrepreneurs set their time threshold individually to maximize their own expected return, the equilibrium time threshold optimally balances between under and over-implementation and maximizes the welfare of the entire economy.

PROPOSITION 7 (TIME THRESHOLD EFFICIENCY). *In a hypothetical economy, in which a social planner could fix* t *to be the time threshold for implementing the project, the welfare of the economy would equal* $W(t) = \sum_{m=1}^{n} \frac{n!}{m!(n-m)!} F(t)^m \left(1 - F(t)\right)^{n-m} (qr_H + m(1-q)r_L)$. *The equilibrium time threshold* t^* *(as specified in Proposition 2) is the unique solution of the optimization problem* $t^* \in argmax_{t \geq 0} W(t)$.

Figure 2 illustrates the welfare $W(t)$ as a function of the time threshold t of implementing the project in a hypothetical economy, in which a social planner can fix the time threshold t of implementing the project. If the time threshold for implementing the project is $t = 0$, then the project is never implemented, so the total expected return in the economy is $W(0) = 0$, as shown in the figure. This extremely severe problem of under-implementation of the project is mitigated by shifting the implementation time threshold upward but at the cost of generating over-implementation. For sufficiently low implementation time thresholds, which belong to the time interval $[0, t^*)$, the probability of under-implementation is still very high while the probability of over-implementation is very low. In this region, the benefit of mitigating the severe under-implementation problem by increasing the time threshold outweighs the cost of exacerbating the relatively minor over-implementation problem. Therefore, as illustrated in the figure, in the time interval $[0, t^*)$, the economy welfare $W(t)$ is increasing in t. Beyond the point of $t = t^*$, the probability of under-implementation is low relative to the probability of over-implementation. As a result, the benefit of further increasing the threshold to reduce the already minor problem of under-implementation is lower than the cost of aggravating the problem of over-investment. Therefore, in the time interval (t^*, ∞), the economy welfare $W(t)$ is decreasing in t, as described in the figure. As formally stated in Proposition 7, and graphically illustrated in Figure 2, the equilibrium time threshold t^* is the point in time at which the function W attains its maximum. Hence, interestingly, although entrepreneurs set their time threshold individually to maximize their own expected return, the equilibrium time threshold t^* nevertheless optimally balances between under and over-implementation, maximizing the total expected return in the economy, and minimizing the welfare distortion.[12]

[Figure 2]

The entrepreneurs in our game are all assumed to be risk-neutral. Accordingly, in making their strategic decisions, they care only about the expected return from their business activities, while being indifferent to the extent to which these activities are risky. It is therefore not surprising that their implementation strategies and the implications of their strategic behavior on the entire economy depend upon the ex-ante expected return $\bar{r}$ from the project. It appears, however, that the implementation strategies that the entrepreneurs adopt in equilibrium also depend on the dispersion of the return distribution. This counterintuitive result is formally presented in Proposition 8.

PROPOSITION 8 (LOSS-AVERSION). *Holding $\bar{r}$ constant, the equilibrium time threshold t^* and the equilibrium welfare $W(t^*)$ are both decreasing in the distance $r_H - r_L$.*

From Proposition 3 it follows that the equilibrium time threshold t^* is increasing in the expected profitability $\bar{r} = qr_H + (1-q)r_L$ of the project, as can be expected. Thus, all else equal, more profitable projects have more time to be executed before the entrepreneurs perceive them as unprofitable. Proposition 8 reveals that the dispersion of the return distribution is also important. When changing the parameters r_H and r_L while keeping the expected return $\bar{r}$ intact, the equilibrium time threshold t^* decreases in the distance $r_H - r_L$.[13] This reflects the notion that the fear of unobserved failures in the past increases the relative weight that entrepreneurs assign to the negative outcome. As a result, an increase in r_H together with a simultaneous decrease in r_L, so that the ex-ante expected return $\bar{r}$ from the project is left unchanged, reduces the ex-post expected return from implementing the project. This means that the dilemma causes the entrepreneurs to act as if they are loss-averse, even though they are risk-neutral and care only about the expected return. Proposition 8 further shows that the resulting equilibrium welfare $W(t^*)$ is also decreasing in the distance $r_H - r_L$, implying that the entrepreneurs' loss aversion has negative implications on the welfare of the entire economy. This means that the dilemma is especially

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[12] This surprising result is in contrast to the common consequences of other types of competitions. For example, in a Cournot competition where competitors set their production quantity individually to maximize their own profit, the aggregated profits of the competitors in the economy could be improved if they could commit to reduce their quantities.

[13] It should be noted that the variance of the random variable $\bar{r}$ is increasing in the distance $r_H - r_L$, but it is not solely determined by $r_H - r_L$. The variance of the random variable $\bar{r}$ is $var[\bar{r}] = q(1-q)(r_H - r_L)$, and it thus determined by both the distance $r_H - r_L$ and the success probability q.

severe in high-risk high-return projects, such as research and development of a new technology or a new medicine.

4. *Extension*

The analysis in the previous section assumes that each entrepreneur necessarily observes the idea at some point in time and the return from the first implementation of the project does not change over time. These assumptions allow us to isolate the effects of the good idea dilemma and analyze them separately from additional forces. In this section, we enrich the good idea game by introducing into it a commonly known expiration date $\bar{t} \in (0, \infty)$, beyond which the project yields a non-positive return. This is a simple way to capture the idea that in a changing economic environment the project is not likely to be profitable forever and to incorporate the impatience of entrepreneurs in the game. Since receiving access to the project after its expiration date is equivalent to not receiving access at all, another interpretation of this assumption is that there is a possibility that the idea would never be observed. The edge case of the extended game, in which $\bar{t} = \infty$, depicts our base game. Proposition 9 establishes the existence and uniqueness of equilibrium in the extended game and characterizes its structure.

PROPOSITION 9 (EQUILIBRIUM). *There exists a unique equilibrium* $(I_1, I_2, \dots, I_n: [0, \infty) \rightarrow \{0,1\})$ *in the extended game. The equilibrium is symmetric and characterized by a positive time threshold* $t^{**} > 0$*, satisfying:* $I_i(t) = \begin{cases} 1 \text{ } if \ \ t < t^{**} \\ 0 \text{ } otherwise \end{cases}$ *for any* $i \in \{1,2, \dots, n\}$ *and* $t \in [0, \infty)$*, where the time threshold* t^{**} *is given by* $t^{**} = \min(t^*, \bar{t})$ *and* t^* *is the time threshold obtained in the equilibrium of the base game (as specified in Proposition 2).*

Proposition 9 shows that the introduction of an expiration date into the game modifies the equilibrium of Proposition 2 straightforwardly. The equilibrium in the extended game differs from the equilibrium in the base game only in that the equilibrium time threshold t^{**} is now the minimum of the endogenous time threshold t^* of Proposition 2 and the exogenous upper bound $\bar{t}$. This implies, that compared to the base game where the time threshold optimally balances under and over-implementation (Proposition 7), here the implementation period $[0, t^{**})$ might be shorter leading to more (less) under (over) implementation than optimal.

Almost all the insights that emerge from the analysis of the base game are robust to the introduction of the expiration date of the project, except for the effect of the number of entrepreneurs on the welfare of the economy.[14] In this context, there is a large debate in the literature about how market concentration affects the incentive of firms to perform research and develop new technologies. Exploring the optimal number of entrepreneurs in innovation races is important, because in some cases regulators have some control over the number of players in an industry. In our base game, where an expiration date is absent, the good idea dilemma introduces an advantage for concentrated industries. A lower number of entrepreneurs in the economy relaxes the dilemma and increases the total expected return in the economy, implying that it is optimal to have only one entrepreneur in the economy. With only one entrepreneur in the economy, the project is always implemented once and the first best welfare is achieved, no matter how far in the future the sole entrepreneur observes the idea. The good idea dilemma completely disappears when there is only one entrepreneur in the economy, because the source of the dilemma is the asymmetry of information between entrepreneurs, which obviously exists only in the presence of at least two entrepreneurs in the economy. Proposition 10 indicates that this result does not carry over to the extended game.[15]

PROPOSITION 10 (OPTIMAL NUMBER OF ENTREPRENEURS). *In the extended game, the optimal number of entrepreneurs that maximizes the economy welfare is decreasing in* $\bar{t}$*, approaching* ∞ *when* $\bar{t}$ *converges to* 0*, and approaching* 1 *when* $\bar{t}$ *converges to* ∞*. The optimal number of entrepreneurs is increasing in* r_H*, increasing in* r_L*, and increasing* q*. For any two cumulative distribution functions* F_1 *and* F_2 *over the support* $[0, \infty)$*, satisfying* $\forall t > 0: F_1(t) < F_2(t)$*, the optimal number of entrepreneurs under* $F = F_1$ *is higher than it is under* $F = F_2$*.*

As shown in Proposition 10, when introducing an expiration date into the good idea game, it is not necessarily optimal anymore to have only one entrepreneur in the economy. This is because, in contrast to the base game where the project's return does not change over time, and it is thus not important when implementation occurs, in the extended game the return becomes non-positive after the expiration

[14] Because in the extended game the time threshold is the minimum of the endogenous time threshold t^* from the base game and the expiration date $\bar{t}$, obviously the analysis of the base game carries over to the extended game when $t^* \leq \bar{t}$. When instead $t^* > \bar{t}$, the time threshold in the extended game does not change anymore with the parameters of the model. As a result, the indirect effect of changes in the modeling parameters is shutdown. Nevertheless, total welfare is still increasing in the profitability parameters. The absence of the indirect effect changes the analysis with respect to the frequency of idea arrival in a straightforward manner. When the frequency is low enough (so that $t^* > \bar{t}$), an increase in the frequency of idea arrival increases the probability of implementation, implying lower (higher) probability of under (over) implementation and a higher total welfare.

[15] For simplicity, we treat the number of entrepreneurs in Proposition 10 as a positive continuous number. This has no effect on the nature of the result. This means that an increase in the continuous number of entrepreneurs is translated into an increase in the discrete number of entrepreneurs only when the increase is substantial enough.

date and thus it is important that the idea would be received early enough. A higher number of entrepreneurs works to increase the probability that the project will be received before the expiration date. This beneficial effect of competition, which does not exist in the base game, implies that the optimal number of entrepreneurs in the economy is not necessarily $n = 1$, and that it is decreasing in the expiration date $\bar{t}$. Proposition 10 further indicates the sensitivity of the optimal number of entrepreneurs to the modeling ingredients r_H, r_L, q, and F. The key behind this analysis is that, as shown in the proof of Proposition 10, the optimal number of entrepreneurs is set sufficiently low that the expiration date $\bar{t}$ is binding, so that in equilibrium $t^{**} = \bar{t}$. As a result, in contrast to the base game where the endogenous time threshold is set in a way that best balances under and over-implementation (Proposition 7), in the extended game the threshold is fixed. Therefore, as the parameters change, disrupting the delicate balance between under and over-implementation, the optimal number of entrepreneurs is adjusted to restore the balance of under and over-implementation. When the profitability parameters r_H, r_L and q increase, the problem of under (over) implementation becomes costlier (less costly). In this case, it is optimal to increase the competition in the economy in order to decrease (increase) the probability of under (over) implementation. Hence, the optimal number of entrepreneurs in the economy is increasing in the profitability parameters r_H, r_L and q. This suggests that receiving early access to the project is more valuable when the project is expected to yield a higher return. As the frequency of the idea arrival, as depicted by the function F, increases, the probability of under (over) implementation decreases (increases). In this case, it is optimal to decrease n in order to reverse this effect. This reflects the notion that the number of entrepreneurs and the frequency of idea arrival work as substitutes in expediting the accessibility of ideas. As a result, when the frequency of idea arrival increases, a simultaneous decrease in the number of entrepreneurs can leave the probability of implementation unchanged. Accordingly, as indicated in Proposition 10, the optimal number of entrepreneurs in the economy is decreasing in the frequency of the idea arrival. Our results may provide policymakers with guidance regarding the impact of government intervention in entrepreneurial activity on social welfare. This is an issue that we believe can be a basis for fruitful future research.

5. Summary and Conclusions

We study the dilemma that usually arises when encountering good ideas for profitable economic initiatives. This is the dilemma of whether to implement a seemingly good idea or fail to act out of fear that the idea is not as good as it seems because otherwise someone else would have already taken advantage of it. This dilemma, which we refer to as the good idea dilemma, is confusing, because the better the idea seems to be, the more acute the wonder of why it has not yet been exploited. We shed light on the confounded nature of the good idea dilemma and explore the dynamics behind the dilemma and its economic implications. The source of the good idea dilemma is the information asymmetry that frequently exists among entrepreneurs with respect to failed past investments. While successful implementations make the project disappear or publicly irrelevant, failures are often privately known only to the entrepreneurs who carried them out. This information asymmetry among entrepreneurs is largely ignored in prior research. By studying how the strategic behavior of entrepreneurs in innovation races is affected by their inability to observe failed past investments of their peers, our study adds to the literature on the skepticism that information asymmetry generates regarding the value of economic endeavors and the resulting disincentive for economic agents to engage in such endeavors (e.g., Akerlof 1970; Capen, Clapp, and Campbell 1971; Rock 1986).

Using a continuous-time innovation game with multiple entrepreneurs, we demonstrate how the uncertainty of the entrepreneurs concerning failed past initiatives of their competitors triggers the good idea dilemma and brings it into their strategic considerations. We show that the good idea dilemma generates doubt that grows over time regarding the profitability of an economic idea, diminishing the incentives of entrepreneurs to implement the idea as time passes. As a result, the best strategy of an entrepreneur upon coming up with an idea or observing an opportunity is to exploit it if and only if access to it was received early enough. The good idea dilemma results in both over-implementation of bad ideas that become accessible to more than one entrepreneur early enough, as well as under-implementation of good ideas that become available to the entrepreneurs only late in the game. We characterize conditions under which bad ideas are more likely to be implemented multiple times and good ideas are more likely to be abandoned, exploring the trade-off between the probability of over-implementation and the probability of under-implementation. We show that, although both over-implementation and under-implementation cause value distortion for the individuals in the economy, the main source of total value distortion is under-implementation. Surprisingly, even though entrepreneurs set their time threshold for implementing the innovative idea individually on purpose to maximize their own expected return, the equilibrium implementation time threshold is shown to be the one that optimally balances between under and over-implementation in a way that minimizes the value distortion in the economy.

Appendix - Proofs

Lemma A, Lemma B and Lemma C establish inequalities that will be useful in the following proofs.

Lemma A. The ratio $R = -\frac{(1-q)r_L}{qr_H}$ satisfies $0 < R < 1$.

Proof of Lemma A. It follows from the assumptions $q \in (0,1)$, $r_H > 0$ and $r_L < 0$ that $R = -\frac{(1-q)r_L}{qr_H} > 0$. It also follows from the assumption $E[\tilde{r}] = qr_H + (1-q)r_L > 0$ that $R = -\frac{(1-q)r_L}{qr_H} < 1$. □

Lemma B. Any scalar $x \in (0,1)$ satisfies $1 - x + x\,ln(x) > 0$.

Proof of Lemma B. For any $x \in (0,1)$, the derivative of the function $g(x) = 1 - x + x\,ln(x)$ is $g'(x) = ln(x) < 0$. Accordingly, the function $g(x)$ is decreasing in x for any $x \in (0,1)$, implying that $g(x) > g(1) = 0$ for any $x \in (0,1)$. □

Lemma C. Any two scalars $x, y \in (0,1)$ satisfy $\frac{1}{(ln(1-x))^2}\left(\frac{ln(1-x)}{x} + \frac{1+ln\left(-y\frac{x}{ln(1-x)}\right)}{1-x}\right) < 0$.

Proof of Lemma C. The expression $\frac{1}{(ln(1-x))^2}\left(\frac{ln(1-x)}{x} + \frac{1+ln\left(-y\frac{x}{ln(1-x)}\right)}{1-x}\right)$ can be rewritten as $\frac{1}{(ln(1-x))^2}\left(\frac{ln(1-x)}{x} + \frac{1+ln(y)+ln(x)-ln(-\ln(1-x))}{1-x}\right)$ or $\frac{(1-x)ln(1-x)+x+xln(y)+xln(x)-xln(-\ln(1-x))}{x(1-x)(ln(1-x))^2}$. Since $x(1-x)(ln(1-x))^2 > 0$ for any $x \in (0,1)$, the sign of $\frac{(1-x)ln(1-x)+x+xln(y)+xln(x)-xln(-\ln(1-x))}{x(1-x)(ln(1-x))^2}$ is the same as the sign of $(1-x)ln(1-x) + x + xln(y) + xln(x) - xln(-\ln(1-x))$. Therefore, we need to show that $(1-x)ln(1-x) + x + xln(y) + xln(x) - xln(-\ln(1-x)) < 0$. It follows from $x, y \in (0,1)$ that $xln(y) < 0$. It is therefore sufficient to prove the following inequality
$(1-x)ln(1-x) + x + xln(x) - xln(-\ln(1-x)) < 0$.
Let $l(x) = (1-x)ln(1-x) + x + xln(x) - xln(-\ln(1-x))$ for any $x \in (0,1)$. The derivative of the function l is $l'(x) = -ln(1-x) + ln(x) + 1 - ln(-\ln(1-x)) + \frac{x}{(1-x)\ln(1-x)}$ or equivalently $l'(x) = ln\left(\frac{x}{-(1-x)\ln(1-x)}\right) - \frac{x}{-(1-x)\ln(1-x)} + 1$. It follows from $x \in (0,1)$ that $1 - x \in (0,1)$ and $\ln(1-x) < 0$. Since $1 - x \in (0,1)$, by Lemma B we have $1 - (1-x) + (1-x)\ln(1-x) > 0$ or equivalently $x > -(1-x)\ln(1-x)$. This inequality is equivalent to $\frac{x}{-(1-x)\ln(1-x)} > 1$ because $1 - x \in (0,1)$ and thus $-(1-x)\ln(1-x) > 0$. Let $s(y) = ln(y) - y + 1$ for $y > 0$. The derivative of the function s is $s'(y) = \frac{1}{y} - 1$, which is positive for $0 < y < 1$ and negative for $y > 1$. Thus, at $y = 1$, $s(y)$

attains its maximum value of $s(1) = 0$. It follows that $s(y) = ln(y) - y + 1 < s(1) = 0$ for any $y > 1$. Hence, $l'(x) = s\left(\frac{x}{-(1-x)\ln(1-x)}\right) = ln\left(\frac{x}{-(1-x)\ln(1-x)}\right) - \frac{x}{-(1-x)\ln(1-x)} + 1 < 0$ for any $x \in (0,1)$. This implies that $l(x) < \lim_{x\to 0} l(x)$ for any $x \in (0,1)$, where $\lim_{x\to 0} l(x) = \lim_{x\to 0}\big(xln(x) - xln(-\ln(1-x))\big) = \lim_{x\to 0} xln\left(\frac{x}{-\ln(1-x)}\right)$. Using L'Hôpital's rule, $\lim_{x\to 0}\frac{x}{-ln(1-x)} = \lim_{x\to 0}\frac{1}{\frac{1}{1-x}} = 1$. Hence, $\lim_{x\to 0} l(x) = \lim_{x\to 0} xln\left(\frac{x}{-\ln(1-x)}\right) = 0 \cdot \ln(1) = 0 \cdot 0 = 0$. We conclude that $l(x) < \lim_{x\to 0} l(x) = 0$ for any $x \in (0,1)$. □

Proof of Lemma 1. For any $i \in \{1,2,\dots,n\}$, the probability that entrepreneur i ascribes to the scenario that at least one other entrepreneur receives access to the project and implements it by time t is given by $\theta_i(t) \equiv 1 - \prod_{j\neq i}(1 - \int_0^t I_j(k)f(k)dk)$. Hence, $(1-q)\theta_i(t)$ is the probability that at least one other entrepreneur has already received the access to the project and implemented it but the implementation resulted in failure, whereas $1 - \theta_i(t)$ is the probability that is that no other entrepreneur has already implemented the project. Using Bayes rule, the probability $\pi_i(t)$ that entrepreneur i attributes at time t to the scenario that at least one other entrepreneur has already experienced failure in implementing the project is given by $\pi_i(t) = \frac{(1-q)\theta_i(t)}{1-\theta_i(t)+(1-q)\theta_i(t)} = \frac{(1-q)\theta_i(t)}{1-q\theta_i(t)}$. At time $t = 0$, $\theta_i(0) = 1 - \prod_{j\neq i}\left(1 - \int_0^0 I_j(k)f(k)dk\right) = 0$, implying $\pi_i(0) = 0$. For any given implementation strategies $(I_1, I_2, \dots, I_n)$, the probability $\theta_i(t)$ is continuous and weakly increasing in t. Hence, the probability $\pi_i(t) = \frac{(1-q)\theta_i(t)}{1-q\theta_i(t)}$ is also continuous and weakly increasing in t. □

Proof of Proposition 2. By Lemma 1, for any $i \in \{1,2,\dots,n\}$ and $t \in [0,\infty)$, the probability $\pi_i(t)$ equals $\pi_i(t) = \frac{(1-q)\theta_i(t)}{1-q\theta_i(t)}$, where $\theta_i(t) \equiv 1 - \prod_{j\neq i}(1 - \int_0^t I_j(k)f(k)dk)$. Therefore, for any $i \in \{1,2,\dots,n\}$ and $t \in [0,\infty)$, the probability $\pi_i(t)$ achieves its maximum across all possible implementation strategies $(I_1, I_2, \dots, I_n)$ when $I_j(k) = 1$ for any $j \neq i$ and $k \in [0,t]$. So, for any $i \in \{1,2,\dots,n\}$ and $t \in [0,\infty)$, $\pi_i(t) \leq \pi(t)$, where $\pi(t) = \frac{(1-q)\left(1-(1-F(t))^{n-1}\right)}{1-q\left(1-(1-F(t))^{n-1}\right)}$. Using the notation $h(t) = \pi(t)r_L + \big(1-\pi(t)\big)\bar{r}$, it follows that $\pi_i(t)r_L + \big(1-\pi_i(t)\big)\bar{r} \geq h(t)$ for any $i \in \{1,2,\dots,n\}$ and $t \in [0,\infty)$. The function $\pi(t)$ is continuous and increasing in t, satisfying $\pi(0) = 0$ and $\lim_{t\to\infty} \pi(t) = 1$. Thus, the function $h(t)$ is continuous and decreasing in t, satisfying $h(0) = \bar{r} > 0$ and $\lim_{t\to\infty} h(t) = r_L < 0$. Therefore, by the intermediate value theorem, there is a unique $t^* \in (0,\infty)$ that solves the equation $h(t^*) = 0$, such that $h(t) > 0$ for any $t < t^*$ and $h(t) < 0$ for any $t > t^*$. Thus, for any $i \in \{1,2,\ldots,n\}$ and $t < t^*$, we get the inequality $\pi_i(t)r_L + \big(1-\pi_i(t)\big)\bar{r} \geq h(t) > 0$, which implies $I_i(t) = 1$. Consequently, for any $i \in \{1,2,\ldots,n\}$ and $t \leq t^*$, $\pi_i(t) = \pi(t)$. Since $\pi_i(t^*)r_L + \big(1-\pi_i(t^*)\big)\bar{r} = 0$, and given that π_i is a non-decreasing function by Lemma 1, it follows that $I_i(t) = 0$ for any $i \in \{1,2,\ldots,n\}$ and $t \geq t^*$, and consequently also $\pi_i(t) = \pi(t^*)$. Substituting $\pi(t^*) = \frac{(1-q)\left(1-(1-F(t^*))^{n-1}\right)}{1-q\left(1-(1-F(t^*))^{n-1}\right)}$ and $\bar{r} = qr_H + (1-q)r_L$ into the equilibrium equation $h(t^*) = \pi(t^*)r_L + \big(1-\pi(t^*)\big)\bar{r} = 0$, and applying simple algebraic rearrangements, we get the equivalent equation $F(t^*) = 1 - (\frac{-(1-q)r_L}{qr_H})^{\frac{1}{n-1}}$. □

Proof of Proposition 3. By Proposition 2, the equilibrium time threshold t^* is the unique solution of the equation $F(t^*) = 1 - R^{\frac{1}{n-1}}$, where $R = \frac{-(1-q)r_L}{qr_H}$. The function $F(t^*)$ on the left hand side of the equilibrium equation is increasing in t^*, while the term $1 - R^{\frac{1}{n-1}}$ on the right hand side of the equation is independent of t^*, increasing in r_H, increasing in r_L, increasing in q, and decreasing in n (as $R \in (0,1)$ by Lemma A). Hence, the solution t^* to the equilibrium equation is increasing in r_H, increasing in r_L, increasing in q, and decreasing in n. For any two cumulative distribution functions F_1 and F_2 over the support $[0,\infty)$, satisfying $\forall t > 0: F_1(t) < F_2(t)$, the left hand side of the equilibrium equation is lower under $F = F_1$ than it is under $F = F_2$, so the solution t^* to the equation is higher under $F = F_1$ than it is under $F = F_2$. □

Proof of Proposition 4. The probability of under-implementation, denoted here P_U, is the probability that no entrepreneur receives access to the project prior to time t^*. It is thus given by $P_U = \big(1-F(t^*)\big)^n$. Since the cumulative distribution function F satisfies $F(t) \in (0,1)$ for any $t \in [0,\infty)$, it follows that $P_U = \big(1-F(t^*)\big)^n$ is positive. By Proposition 2, $F(t^*) = 1 - R^{\frac{1}{n-1}}$, where $R = \frac{-(1-q)r_L}{qr_H}$. Hence, the probability of under-implementation can be rewritten as $P_U = \big(1-F(t^*)\big)^n = R^{\frac{n}{n-1}}$. As $R = \frac{-(1-q)r_L}{qr_H}$ is decreasing in r_H, decreasing in r_L, decreasing in q, and independent of F, so is also $P_U = R^{\frac{n}{n-1}}$. Also, since $R \in (0,1)$ by Lemma A and $\frac{n}{n-1}$ is decreasing in n, it follows that $P_U = R^{\frac{n}{n-1}}$ is increasing in n. □

Proof of Proposition 5. The probability of over-implementation, denoted here P_O, is the probability that the project yields the negative return r_L and two or more entrepreneurs receive access to the project prior to time t^*. It is therefore given by $P_O = (1-q)\sum_{m=2}^{n} \frac{n!}{m!(n-m)!} F(t^*)^m \left(1-F(t^*)\right)^{n-m}$ or equivalently $P_O = (1-q)\left(1-\left(1-F(t^*)\right)^n - nF(t^*)\left(1-F(t^*)\right)^{n-1}\right)$. Using now the equilibrium equation $F(t^*) = 1 - R^{\frac{1}{n-1}}$ given in Proposition 2, the probability of over-implementation can be equivalently rewritten as $P_O = (1-q)\left(1-nR+(n-1)R^{\frac{n}{n-1}}\right)$. Hence, P_O is independent of F. The derivative of P_O with respect to R is $\frac{dP_O}{dR} = -(1-q)n\left(1-R^{\frac{1}{n-1}}\right) < 0$, implying that P_O is decreasing in R. Since R equals $\frac{-(1-q)r_L}{qr_H}$, and is decreasing in r_H and in r_L, it follows that P_O is increasing in r_H and in r_L. The derivative of P_O with respect to n is $\frac{dP_O}{dn} = (1-q)\left(-R + R^{\frac{n}{n-1}}\left(1-\frac{\ln(R)}{n-1}\right)\right)$. The derivative $\frac{dP_O}{dn}$ is increasing in n, because $\frac{d^2P_O}{dn^2} = (1-q)\frac{R^{\frac{n}{n-1}}\ln(R)^2}{(n-1)^3} > 0$. At $n=2$, the derivative attains the value of $-(1-q)R(1-R+R\ln(R))$, which is negative because $1-R+R\,ln(R) > 0$ by Lemma A and Lemma B. Also, $\lim_{n\to\infty}\frac{dP_O}{dn} = 0$. We therefore conclude that $\frac{dP_O}{dn} < 0$ for any $n \geq 2$, implying that P_O is decreasing in n. It follows from the assumption $qr_H + (1-q)r_L > 0$ that $q > \frac{-r_L}{r_H - r_L}$. The derivative of P_O with respect to q is $\frac{dP_O}{dq} = -1 + n\frac{r_L}{r_H} - n\frac{r_L}{q^2 r_H} - R^{\frac{n}{n-1}}\left(\frac{n}{q}+n-1\right)$, where $\lim_{q\to\frac{-r_L}{r_H-r_L}}\frac{dP_O}{dq} = 0$ and $\lim_{q\to 1}\frac{dP_O}{dq} = -1$. The second derivative of P_O with respect to q is $\frac{d^2P_O}{dq^2} = \frac{n}{q^3}\left(\frac{2r_L}{r_H} + \frac{2n-1}{n-1}\frac{q}{1-q}R^{\frac{n}{n-1}}\right)$, which has the sign of $k(q) = \frac{2r_L}{r_H} + \frac{2n-1}{n-1}\frac{q}{1-q}R^{\frac{n}{n-1}}$, where $k'(q) = -\frac{2n-1}{(n-1)^2(1-q)^2}R^{\frac{n}{n-1}} < 0$, $\lim_{q\to\frac{-r_L}{r_H-r_L}} k(q) = \frac{-r_L}{r_H(n-1)} > 0$ and $\lim_{q\to 1} k(q) = \frac{2r_L}{r_H} < 0$. It follows that the second derivative $\frac{d^2P_O}{dq^2}$ is positive for sufficiently low values of q, attaining zero, and then becomes negative for higher values of q. Accordingly, the first derivative $\frac{dP_O}{dq}$ is initially increasing in q from zero (at $q = \frac{-r_L}{r_H-r_L}$), attaining a positive maximum, and then decreasing in q up to the negative value of -1 (at $q=1$). This implies that P_O is non-monotonic in q – initially increasing in q, attaining a maximum, and then decreasing in q. □

Proof of Proposition 6. If none of the entrepreneurs receives the access to the project by time t^*, then the welfare in the economy is zero. For any $m = 1,2,\ldots,n$, if exactly m entrepreneurs receive the access to the project prior to time t^*, then with probability q the project is implemented only once yielding the positive return r_H and with probability $1-q$ the project is implemented m times yielding a negative return r_L. So, if exactly $m \geq 1$ entrepreneurs receive the access to the project prior to time t^*, then the expected welfare in the economy is $qr_H + (1-q)mr_L$. The probability that exactly m entrepreneurs receive the access to the project by time t^* is $\frac{n!}{m!(n-m)!}F(t^*)^m\big(1-F(t^*)\big)^{n-m}$. So, the ex-ante welfare is $W(t^*) = \sum_{m=1}^{n}\frac{n!}{m!(n-m)!}F(t^*)^m\big(1-F(t^*)\big)^{n-m}(qr_H + (1-q)mr_L)$. The welfare $W(t^*)$ can be rewritten as $qr_H\big(1-\big(1-F(t^*)\big)^n\big) + n(1-q)r_L F(t^*)\sum_{m=1}^{n}\frac{(n-1)!}{(m-1)!(n-m)!}F(t^*)^{m-1}\big(1-F(t^*)\big)^{n-m}$ or as $qr_H\big(1-\big(1-F(t^*)\big)^n\big) + n(1-q)r_L F(t^*)\sum_{m=0}^{n-1}\frac{(n-1)!}{m!(n-1-m)!}F(t^*)^{m}\big(1-F(t^*)\big)^{n-1-m}$, which equals $qr_H\big(1-\big(1-F(t^*)\big)^n\big) + n(1-q)r_L F(t^*)$. Using the equilibrium equation $F(t^*) = 1 - R^{\frac{1}{n-1}}$ given in Proposition 2, the welfare $W(t^*)$ can be rewritten as $qr_H\left(1-R^{\frac{n}{n-1}}\right) + n(1-q)r_L\left(1-R^{\frac{1}{n-1}}\right)$ or $qr_H - qr_H R^{\frac{n}{n-1}} + n(1-q)r_L\left(1-R^{\frac{1}{n-1}}\right)$. Substituting $qr_H = -\frac{(1-q)r_L}{R}$, we get that the welfare $W(t^*)$ equals $qr_H + (1-q)r_L R^{\frac{1}{n-1}} + n(1-q)r_L\left(1-R^{\frac{1}{n-1}}\right)$ or $qr_H + (1-q)r_L\left(n-(n-1)R^{\frac{1}{n-1}}\right)$. So, $W(t^*) = qr_H + (1-q)r_L + (1-q)r_L(n-1)\left(1-R^{\frac{1}{n-1}}\right) < qr_H + (1-q)r_L = \bar{r}$. Also, $W(t^*)$ is independent of F. The derivative of $W(t^*)$ with respect to r_H is $\frac{dW(t^*)}{dr_H} = q - (1-q)r_L R^{-\frac{n-2}{n-1}}\cdot\frac{dR}{dr_H}$ or equivalently $\frac{dW(t^*)}{dr_H} = q\left(1-R^{\frac{n}{n-1}}\right)$ after substituting $\frac{dR}{dr_H} = \frac{(1-q)r_L}{q{r_H}^2} = \frac{qR^2}{(1-q)r_L}$. As $R \in (0,1)$ by Lemma A, it follows that $\frac{dW(t^*)}{dr_H} = q\left(1-R^{\frac{n}{n-1}}\right) > 0$, and thus $W(t^*)$ is increasing in r_H. The derivative of $W(t^*)$ with respect to r_L is $\frac{dW(t^*)}{dr_L} = (1-q)\left(n-(n-1)R^{\frac{1}{n-1}}\right) - (1-q)r_L R^{-\frac{n-2}{n-1}}\cdot\frac{dR}{dr_L}$ or equivalently $\frac{dW(t^*)}{dr_L} = (1-q)n\left(1-R^{\frac{1}{n-1}}\right)$ after substituting $\frac{dR}{dr_L} = \frac{-(1-q)}{qr_H} = \frac{R}{r_L}$. As $R \in (0,1)$ by Lemma A, it follows that $\frac{dW(t^*)}{dr_L} = (1-q)n\left(1-R^{\frac{1}{n-1}}\right) > 0$, and thus $W(t^*)$ is increasing in r_L. The derivative of $W(t^*)$ with respect to q is $\frac{dW(t^*)}{dq} = r_H - r_L\left(n-(n-1)R^{\frac{1}{n-1}}\right) - (1-q)r_L R^{-\frac{n-2}{n-1}}\cdot\frac{dR}{dq}$ or equivalently $\frac{dW(t^*)}{dq} = r_H + r_L\left(\left(n-1+\frac{1}{q}\right)R^{\frac{1}{n-1}} - n\right)$ after substituting $\frac{dR}{dq} = \frac{r_L}{q^2 r_H} = \frac{R}{-q(1-q)}$. The second derivative of $W(t^*)$ with respect to q is $\frac{d^2W(t^*)}{dq^2} = -r_L \frac{1}{q^2} R^{\frac{1}{n-1}} + r_L \frac{1}{n-1}\left(n-1+\frac{1}{q}\right) R^{-\frac{n-2}{n-1}} \cdot \frac{dR}{dq}$ or equivalently $\frac{d^2W(t^*)}{dq^2} = -\frac{nr_L R^{\frac{1}{n-1}}}{(n-1)(1-q)q^2}$ after substituting $\frac{dR}{dq} = \frac{r_L}{q^2 r_H} = \frac{R}{-q(1-q)}$. As $\frac{d^2W(t^*)}{dq^2} = -\frac{nr_L R^{\frac{1}{n-1}}}{(n-1)(1-q)q^2} > 0$, it follows that the first derivative $\frac{dW(t^*)}{dq}$ is increasing in q. Since $\lim_{q \to \frac{-r_L}{r_H - r_L}} \frac{dW(t^*)}{dq} = 0$ and $\lim_{q \to 1} \frac{dW(t^*)}{dq} = r_H - nr_L > 0$, it further follows that $\frac{dW(t^*)}{dq} > 0$ for any $q \in (\frac{-r_L}{r_H - r_L}, 1)$, implying that $W(t^*)$ is increasing in q. The derivative of $W(t^*)$ with respect to n is $\frac{dW(t^*)}{dn} = (1-q)r_L\left(1 + R^{\frac{1}{n-1}}\left(\frac{\ln(R)}{n-1} - 1\right)\right)$. The second derivative of $W(t^*)$ with respect to n is $\frac{d^2W(t^*)}{dn^2} = -(1-q)r_L R^{\frac{1}{n-1}} \frac{\ln(R)^2}{(n-1)^3} > 0$, and thus the first derivative $\frac{dW(t^*)}{dn}$ is increasing in n. At n=2, $\frac{dW(t^*)}{dn}$ equals $(1-q)r_L(1 - R + R\ln(R))$, which is negative by Lemma A and Lemma B. Also, $\lim_{n \to \infty} \frac{dW(t^*)}{dn} = 0$. Hence, the first derivative $\frac{dW(t^*)}{dn}$ is negative for any $n \geq 2$, implying that $W(t^*)$ is decreasing in n. □

Proof of Proposition 7. Using the same arguments as in the proof of Proposition 6, the welfare in a hypothetical economy, in which t is the time threshold for implementing the project, is given by $W(t) = \sum_{m=1}^{n} \frac{n!}{m!(n-m)!} F(t)^m \left(1 - F(t)\right)^{n-m} (qr_H + m(1-q)r_L)$ and it can be equivalently rewritten as $W(t) = qr_H\left(1 - \left(1 - F(t)\right)^n\right) + n(1-q)r_L F(t)$. Let the function $U: [0,1] \to \Re$ be defined as $U(x) = qr_H(1 - (1-x)^n) + n(1-q)r_L x$ and note that $W(t) = U(F(t))$. In order to find $argmax_{x \in [0,1]} U(x)$, note that $U'(x) = qr_H n(1-x)^{n-1} + n(1-q)r_L$. Hence, the first order condition is $U'(x) = qr_H n(1-x)^{n-1} + n(1-q)r_L = 0$ or equivalently $x = 1 - R^{\frac{1}{n-1}}$. The second order condition is $U''(x) = -qr_H n(n-1)(1-x)^{n-2} < 0$. Therefore, the function $U(x)$ attains its maximum at $x = 1 - R^{\frac{1}{n-1}}$. As $W(t) = U(F(t))$, it follows that the function $W(t)$ attains its maximum at the solution to the equation $F(t) = 1 - R^{\frac{1}{n-1}}$, which is t^* by Proposition 2. □

Proof of Proposition 8. Holding $\bar{r} = qr_H + (1-q)r_L$ constant, r_L can be represented as $r_L = \frac{\bar{r} - qr_H}{1-q}$, and thus $r_H - r_L$ can be represented as $r_H - r_L = r_H - \frac{\bar{r} - qr_H}{1-q} = \frac{r_H - \bar{r}}{1-q}$. Therefore, when $\bar{r}$ is kept intact, any change of Δ in r_H must be associated with a change in the opposite direction of $-\frac{q}{1-q}\Delta$ in r_L in and with change in the same direction of $\frac{1}{1-q}\Delta$ in $r_H - r_L$. Using $r_L = \frac{\bar{r}-qr_H}{1-q}$, the ratio R can be represented as $R = -\frac{(1-q)r_L}{qr_H} = -\frac{\bar{r}-qr_H}{qr_H}$, and accordingly the equilibrium equation $F(t^*) = 1 - R^{\frac{1}{n-1}}$ from Proposition 2 can be rewritten as $F(t^*) = 1 - (-\frac{\bar{r}-qr_H}{qr_H})^{\frac{1}{n-1}}$. Since the cumulative distribution function F on the left hand side of this equation is an increasing function and the expression $1 - (-\frac{\bar{r}-qr_H}{qr_H})^{\frac{1}{n-1}}$ on the right hand side of the equation is decreasing in r_H, the solution t^* to the equation is decreasing in r_H, and it is thus also decreasing in $r_H - r_L$. Employing the equality $r_L = \frac{\bar{r}-qr_H}{1-q}$ again on the welfare formula $W(t^*) = qr_H + (1-q)r_L\left(n-(n-1)R^{\frac{1}{n-1}}\right)$ from the proof of Proposition 6, we get the equivalent formula $W(t^*) = qr_H + (\bar{r}-qr_H)\left(n-(n-1)\left(-\frac{\bar{r}-qr_H}{qr_H}\right)^{\frac{1}{n-1}}\right)$. The derivative of $W(t^*)$ with respect to r_H is $-q\left(n\left(1-\left(-\frac{\bar{r}-qr_H}{qr_H}\right)^{\frac{1}{n-1}}\right)-\left(1-\left(-\frac{\bar{r}-qr_H}{qr_H}\right)^{\frac{n}{n-1}}\right)\right)$ or $-q\left(n\left(1-R^{\frac{1}{n-1}}\right)-\left(1-R^{\frac{n}{n-1}}\right)\right)$. Since $R \in (0,1)$ by Lemma A, it follows that $R^{\frac{n}{n-1}} < R^{\frac{n1}{n-1}}$. We consequently get the inequality $-q\left(n\left(1-R^{\frac{1}{n-1}}\right)-\left(1-R^{\frac{n}{n-1}}\right)\right) < -q(n-1)\left(1-R^{\frac{1}{n-1}}\right) < 0$. Therefore, the welfare $W(t^*)$ is decreasing in r_H and in $r_H - r_L$. □

Proof of Proposition 9. In the case where $t^* \leq \bar{t}$, it follows from the proof of Proposition 2 that $\pi_i(t^*)r_L + \left(1-\pi_i(t^*)\right)\bar{r} = 0$ for any $i \in \{1,2,\ldots,n\}$, and thus it must be that $t^{**} = t^*$. When instead $t^* > \bar{t}$, then $\bar{\pi}(t)r_L + \left(1-\bar{\pi}(t)\right)\bar{r} > 0$ for any $i \in \{1,2,\ldots,n\}$ and $t \in [0,\bar{t}]$, so that implementation of the project occurs throughout the entire period $[0,\bar{t}]$ and accordingly $t^{**} = \bar{t}$. □

Proof of Proposition 10. In the base game, the time threshold t^* is decreasing in n by Proposition 3. Hence, for sufficiently low values of n, the expiration date is binding so that $t^{**} = \bar{t}$ and the welfare is $W(\bar{t})$, while for sufficiently high values n of the expiration date is not binding so that $t^{**} = t^*$ and the welfare is $W(t^*)$. By Proposition 6, the welfare $W(t^*)$ is decreasing in n. It thus follows that the optimal n is sufficiently low that the expiration date $\bar{t}$ is binding and the welfare is $W(\bar{t})$. As a result, under the optimal n, in equilibrium $t^{**} = \bar{t}$. By the proof of Proposition 7, the welfare in this case is $W(\bar{t}) =$

$qr_H\left(1-\left(1-F(\bar{t})\right)^n\right)+n(1-q)r_L F(\bar{t})$. The derivative of $W(\bar{t})$ with respect to n is $\frac{dW(\bar{t})}{dn} = -q\left(1-F(\bar{t})\right)^n ln\left(1-F(\bar{t})\right)r_H + (1-q)F(\bar{t})r_L$. So, the first order condition is $\frac{dW(\bar{t})}{dn} = -q\left(1-F(\bar{t})\right)^n ln\left(1-F(\bar{t})\right)r_H + (1-q)F(\bar{t})r_L = 0$ or equivalently n= $\frac{ln\left(-R\frac{F(\bar{t})}{ln(1-F(\bar{t}))}\right)}{ln\left(1-F(\bar{t})\right)}$, and the second order condition is $\frac{d^2W(\bar{t})}{dn^2} = -q\left(1-F(\bar{t})\right)^n \left(ln\left(1-F(\bar{t})\right)\right)^2 r_H < 0$. The optimal number of entrepreneurs is therefore $\frac{ln\left(-R\frac{F(\bar{t})}{ln(1-F(\bar{t}))}\right)}{ln\left(1-F(\bar{t})\right)}$. To ensure consistency, we need to verify that this optimal solution indeed implies $t^{**} = \bar{t}$. For that, it should be noted that t^* is decreasing in n by Proposition 3, and the highest level of n that yields $t^{**} = \bar{t}$ satisfies $F(t^*) = 1 - R^{\frac{1}{n-1}} = F(\bar{t})$, implying that it is equal to $\frac{ln\left(R\left(1-F(\bar{t})\right)\right)}{ln\left(1-F(\bar{t})\right)}$. By Lemma B, for any $R, F(\bar{t}) \in (0,1)$, we have $F(\bar{t}) + ln\left(1-F(\bar{t})\right)\left(1-F(\bar{t})\right) > 0$ or equivalently $-\frac{F(\bar{t})}{ln\left(1-F(\bar{t})\right)} > 1 - F(\bar{t})$, and thus $\frac{ln\left(-R\frac{F(\bar{t})}{ln(1-F(\bar{t}))}\right)}{ln\left(1-F(\bar{t})\right)} < \frac{ln\left(R\left(1-F(\bar{t})\right)\right)}{ln\left(1-F(\bar{t})\right)}$ implying $t^{**} = \bar{t}$. The derivative of $\frac{ln\left(-R\frac{F(\bar{t})}{ln(1-F(\bar{t}))}\right)}{ln\left(1-F(\bar{t})\right)}$ with respect to $\bar{t}$ equals $\frac{f(\bar{t})}{\ln\left(1-F(\bar{t})\right)^2} \cdot \left(\frac{ln\left(1-F(\bar{t})\right)}{F(\bar{t})} + \frac{1+ln\left(-R\frac{F(\bar{t})}{ln(1-F(\bar{t}))}\right)}{1-F(\bar{t})}\right)$, and it is negative by Lemma A and Lemma C, implying that the optimal number of entrepreneurs is decreasing in $\bar{t}$. Using L'Hopital rule, $\lim_{\bar{t}\to 0} \frac{ln\left(-R\frac{F(\bar{t})}{ln(1-F(\bar{t}))}\right)}{ln\left(1-F(\bar{t})\right)} = \infty$ and $\lim_{\bar{t}\to \infty} \frac{ln\left(-R\frac{F(\bar{t})}{ln(1-F(\bar{t}))}\right)}{ln\left(1-F(\bar{t})\right)} = 0$, suggesting that the optimal number of entrepreneurs approaches ∞ when $\bar{t}$ converges to 0 and approaches 1 when $\bar{t}$ converges to ∞. Denoting $F(\bar{t})$ by x, the derivative of $\frac{ln\left(-R\frac{x}{ln(1-x)}\right)}{ln(1-x)}$ with respect to x is $\frac{1}{\ln(1-x)^2} \cdot \left(\frac{ln(1-x)}{x} + \frac{1+ln\left(-R\frac{x}{ln(1-x)}\right)}{1-x}\right)$, which is negative for any $x \in (0,1)$ by Lemma A and Lemma C. Hence, for any two cumulative distribution functions F_1 and F_2 over the support $[0, \infty)$, satisfying $\forall t > 0: F_1(t) < F_2(t)$, the optimal number of entrepreneurs under $F = F_1$ is higher than it is under $F = F_2$. The derivative of $\frac{ln\left(-R\frac{F(\bar{t})}{ln(1-F(\bar{t}))}\right)}{ln\left(1-F(\bar{t})\right)}$ with respect to R equals $\frac{1}{Rln\left(1-F(\bar{t})\right)}$, and it is negative because $F(\bar{t}) \in (0,1)$. This implies that the optimal number of entrepreneurs is decreasing in the ratio R, and it is thus increasing in r_H, increasing in r_L, and increasing in q. □

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Figures

Figure 1

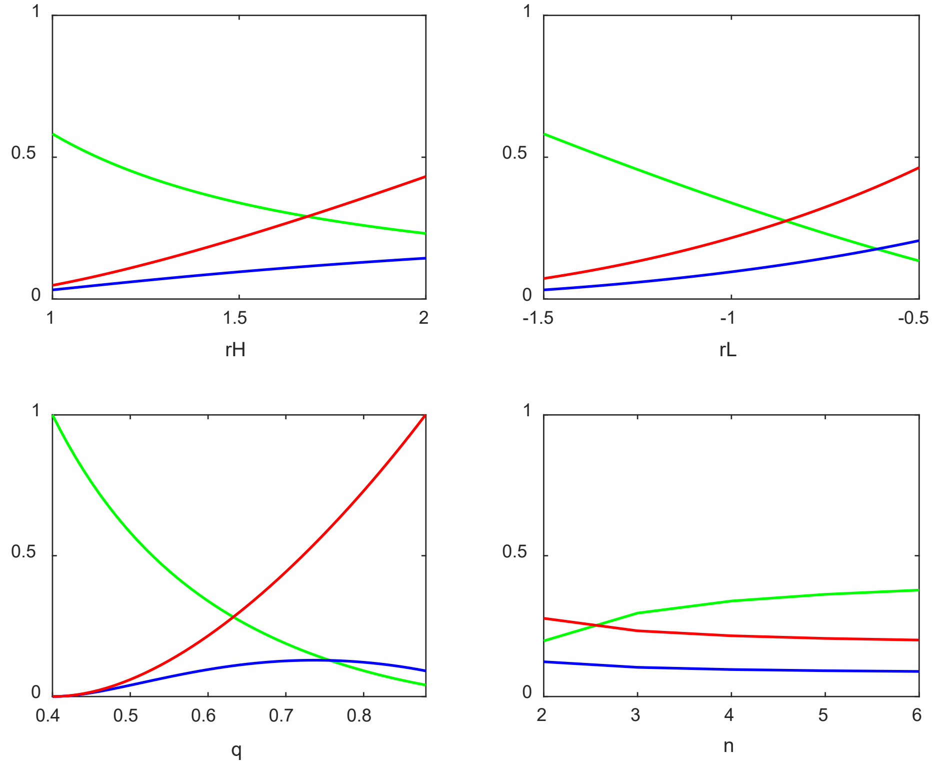

Figure 1 illustrates the sensitivity of the main equilibrium outcomes to the modeling parameter. In all four plots, three equilibrium outcomes are illustrated: the probability of under-implementation in green, the probability of over-implementation in blue, and the welfare in red. The top left plot pertains to the special case of $r_L = -1$, $q = 0.6$, $n = 4$, and it describes the three equilibrium outcomes as a function of the parameter r_H that varies in the interval $(1,2)$. The top right plot pertains to the special case of $r_H = 1.5$, $q = 0.6$, $n = 4$, and it describes the three equilibrium outcomes as a function of the parameter r_L that varies in the interval $(-1.5, -0.5)$. The bottom left plot pertains to the special case of $r_L = -1$, $r_H = 1.5$, $n = 4$, and it describes the three equilibrium outcomes as a function of the parameter q that varies in the interval $(0.4, 0.9)$. The bottom right plot pertains to the special case of $r_L = -1$, $r_H = 1.5$, $q = 0.6$, and it describes the three equilibrium outcomes as a function of the parameter n that varies in the interval $(2,6)$.

Figure 2

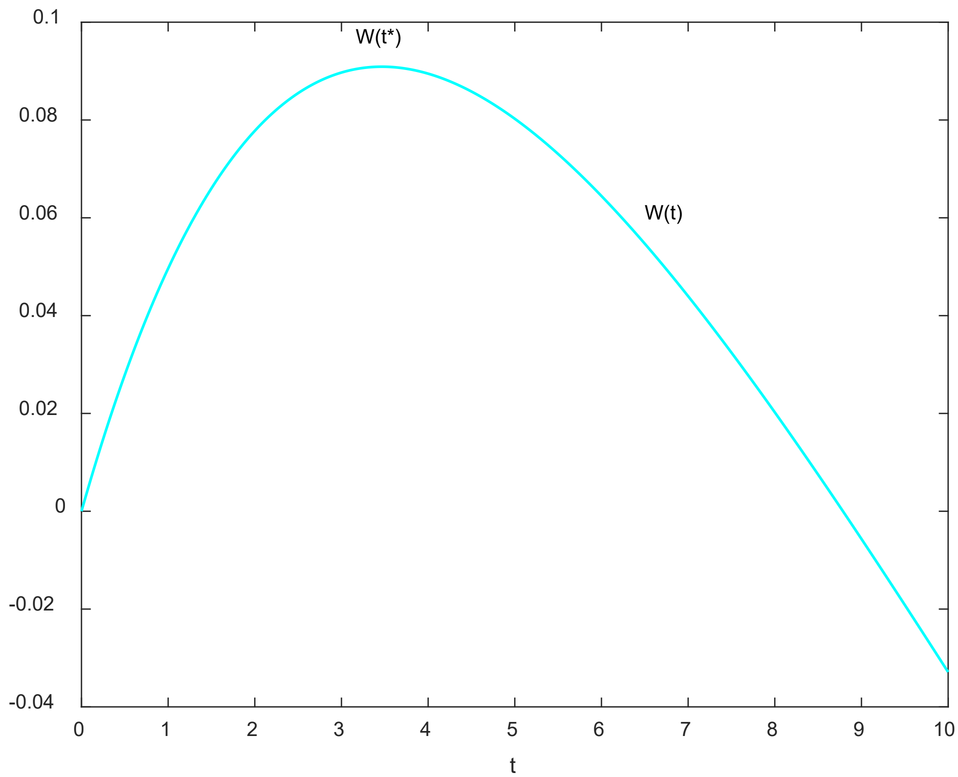

Figure 2 pertains to the special case where $r_L = -0.5$, $r_H = 1$, $q = 0.5$, $n = 5$, and an exponential distribution of the random variables $\tilde{t}_1, \tilde{t}_2, \dots, \tilde{t}_n \sim Exp(\lambda)$ with the arrival rate parameter $\lambda = 0.05$, so that the cumulative distribution function is $F(t) = 1 - exp(-\lambda t)$. The figure illustrates the welfare of the economy, $W(t)$, as a function of the time threshold t of implementing the project. The equilibrium time threshold t^* is the point in time at which the function W attains its maximum.